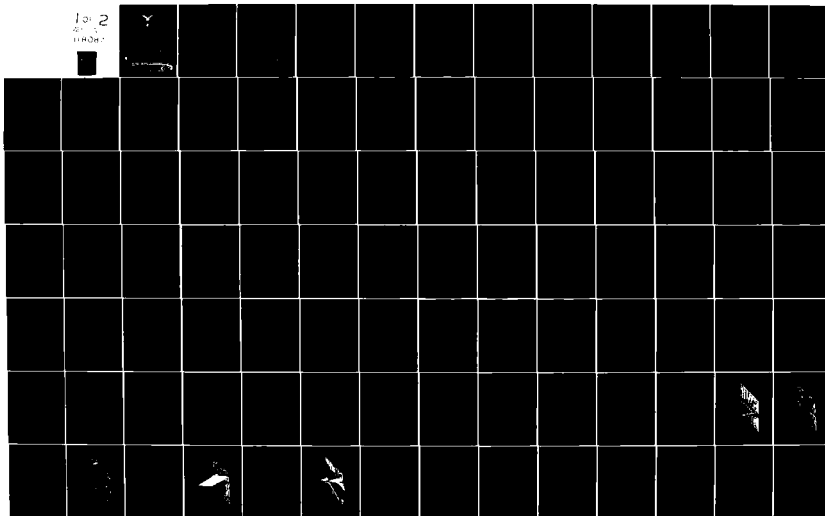


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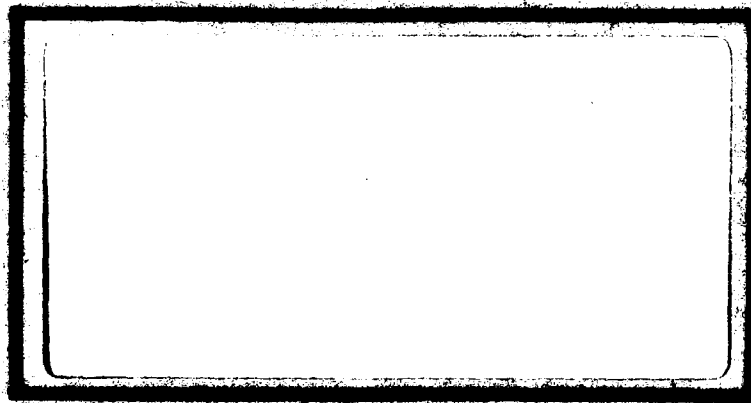
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RADAR WAVEFORM SELECTION BASED ON  
THE CALCULATION AND APPLICATION  
OF RADAR AMBIGUITY FUNCTIONS

Thesis

AFIT/GE/EE/82J-11

John D. Reed  
Capt USAF

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RADAR WAVEFORM SELECTION BASED ON  
THE CALCULATION AND APPLICATION  
OF RADAR AMBIGUITY FUNCTIONS

THESIS

Presented to the Faculty of the School of Engineering  
of the Air Force Institute of Technology  
Air University  
in Partial Fulfillment of the  
Requirements for the Degree of  
Master of Science

by

John D. Reed, BS, BSEE  
Capt USAF

Graduate Electrical Engineering

June 1982

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John D. Reed

## Preface

The purpose of this study was to derive a method of selecting a radar waveform which would satisfy a given set of criteria for target detection in a specified environment. The method derived in this study was to calculate and graphically display the magnitude of the radar ambiguity function for radar waveforms which met some general criteria. After analyzing these magnitude displays, those which approximated the resolution requirements for target detection were then analyzed for their performance in the specified environment. To assist in this analysis, the signal-to-clutter ratio for the desired target in a model of the clutter environment was calculated. The waveform which best met the criteria for target detection in the modeled environment was then determined.

A limited group of radar waveforms were investigated in this study; however, all waveform types generally used in contemporary radars were investigated. The ambiguity functions were calculated as a function of range delay and Doppler frequency. Calculations of ambiguity functions have also been made as a function of azimuth angle and elevation angle, but these are not significant in selecting the radar waveform and were not considered.

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Abstract

The magnitude of the radar ambiguity function for commonly used radar waveforms is calculated and displayed in three-dimensional diagrams. The ambiguity functions are calculated as a function of range delay and Doppler frequency. For a specified clutter environment and a particular radar waveform, the signal-to-clutter ratio for a desired target is calculated. A method to select a radar waveform, given the desired target detection criteria and clutter environment, is derived. This method relies on the analysis of the magnitude of the ambiguity function, signal-to-clutter ratio, and other basic principles of radar theory.

## I. Introduction

### Statement of the Problem

A significant problem encountered by a radar engineer is the choice of a radar waveform which provides the desired detection of targets in some given environment. This problem is one of determining which waveform can provide the desired resolution in both range and Doppler, the desired detection of targets, minimal ambiguities in the measurement of target parameters, and satisfactory clutter-rejection properties for the specified environment (Ref 11:420). The selected waveform can only be implemented if it can be transmitted and received within the size and budget constraints of the system. By calculating the ambiguity functions of radar waveforms and analyzing the data from them, the radar engineer would have most of the information he needs to make such a determination. If he also could determine the signal-to-clutter ratio for the desired target in the specified environment, he would have all the necessary tools to solve the problem.

Providing a fast, simple, and accurate method for selecting a radar waveform which satisfies the target detection requirements in a given environment is the purpose of this study. This method relies on the ability to calculate the magnitude of ambiguity functions for various waveforms, calculate the signal-to-clutter ratio for the specified target and environment, and analyze these results.

## Background

In Gaussian noise, the signal ambiguity function plays an important part in obtaining a maximum likelihood estimate of signal parameters. In fact, in white noise, the estimate is obtained by constructing the ambiguity function as a function of the parameters to be estimated and the values which maximize the function are taken as the desired estimate. Woodward has shown how this is done for delay and Doppler and has derived formulas for the variances of the estimates. For more general maximum likelihood estimates, Kelly, Reed, and Root introduced a more general version of the ambiguity function, which in the case of white Gaussian noise is essentially proportional to the Woodward ambiguity function if the unknown parameters are delay and Doppler (Ref 14:39).

Since the concept of the ambiguity function was first introduced in 1953, a multitude of research has been performed in the field. The ambiguity functions for a large number of waveforms have been calculated, and the results are well documented. Several authors have provided methods of selecting a radar waveform based on analysis of ambiguity functions. Of these, some have provided only limited criteria for the range and velocity resolution for a waveform without much regard to the clutter environment (e.g. Sinsky) (Ref 10). Others have attempted to synthesize ambiguity functions to approximate a function which would provide the optimum performance in the clutter environment and in resolution capabilities (e.g.

Ref 13). The problem is that the techniques suggested up to now are either too simple to provide accuracy or too complicated to provide the radar engineer with a fast method to choose a radar waveform.

#### Approach to Solving the Problem

The method to choose a radar waveform suggested in this study is based on the ability to quickly and accurately calculate radar ambiguity functions, graphically display the ambiguity function magnitude, and calculate the signal-to-clutter ratio. The mathematics required to perform the necessary calculations dictated the use of a computing system with a high-level language capability, a main memory of at least 32K bytes, access to a Fast Fourier transform program, and a method of providing three-dimensional diagrams. During this study, the Data General Eclipse computer was used since it met all the necessary requirements. Fortran V was the programming language selected because it provided the fastest and most efficient means to perform the calculations on the Eclipse.

In order to calculate the ambiguity function, the approach in this study is to take cuts of the ambiguity function for various values of Doppler. With this approach, each cut of the ambiguity function is calculated by performing a linear convolution of appropriately transformed samples of the waveform. The magnitude of each sample is then calculated.

By appropriately spacing these Doppler cuts, an accurate representation of the magnitude of the ambiguity function is provided. This approach also allows simple application to three-dimensional display techniques. Since the magnitude data are sampled in both delay and Doppler, the data can be easily formatted for use by most available three-dimensional plotting programs.

In addition to the magnitude of the radar ambiguity function, the calculation of the signal-to-clutter ratio also has several other required inputs. The specified environment must be modeled by a probability density function of clutter as a function of delay and Doppler. Additionally, the average radar cross section of the desired target and the clutter must be determined as well as the number of different scatterers providing radar returns in the environment. With these inputs, an accurate approximation of the signal-to-clutter ratio is calculated.

The method of radar waveform selection in this study considers the range and velocity resolution requirements, blind speed limitations, unambiguous range requirement, and the signal-to-clutter ratio requirements for the radar system. With the capability to calculate and display the ambiguity function magnitude and to calculate the signal-to-clutter ratio, all of the selection criteria can be evaluated.



### Limitations

This study was limited in the types of radar waveforms considered, the ambiguity function variables considered, and the amount of target motion allowed. To begin, only those radar waveforms generally used in contemporary radars were considered. These waveforms included: simple pulse, pulse with linear frequency modulation (FM), pulse with V-FM, phase-shift coded pulse, and repeated pulses. The majority of waveforms ever seriously considered for current use are contained in this group.

Although a general version of the ambiguity function is available, the parameters used in radar ambiguity functions are range delay, Doppler, azimuth angle, and elevation angle. Since the ambiguities of azimuth and elevation angles effect the antenna design of the system, they must be considered by the radar engineer. Those angles, however, do not significantly affect the waveform selection process in the narrowband case and are not considered in this study (Ref 15:2104).

In addition to the above limitations, this study considers only the case where there is negligible distortions of the complex envelope of the waveform by target motion, target acceleration, and higher order derivatives. This case does, however, include a wide range of practical applications. When this is not the case, the resolution theory becomes much more complex. The theory has been extended by many authors including Rihaczek (Ref 5:Chapter 11 and Ref 8).

## Presentation

To provide some insight into the method of selecting a waveform, this study begins by discussing the significant theory associated with the radar ambiguity function. The concept of the complex signal representation of a radar waveform is discussed. The development of the matched-filter receiver and the optimum receiver in white Gaussian noise is then presented. Properties of the ambiguity function are considered as well as the uncertainty relation between delay and Doppler. A short discussion of the digital signal processing techniques used to compute the ambiguity function is included. Some characteristics of clutter for particular environments and some effects of clutter on detection are also presented.

This study continues with a discussion of the software developed to help resolve the problem of waveform selection. The method of calculating cuts of the ambiguity function for various Doppler frequencies is explained. Use of the three-dimensional graphic technique to produce diagrams of the magnitude as a function of delay and Doppler is discussed. The requirements and method used to calculate the signal-to-clutter ratio ends this section of the study.

After generating the software products, a methodology of analyzing them is needed. This study provides a discussion of a methodology and how it was developed. Some general

conclusions on how to select a waveform and some recommendations for further development of the software end this study.

## II. Theoretical Considerations

### Complex Signal Representation

Radar signals consist of a high-frequency carrier modulated in amplitude or phase by a function that varies much slower than the carrier frequency. These signals can be represented by the equation:

$$S(t) = M(t) \cos (\omega_0 t + \phi(t)) \quad (1)$$

where:

$M(t)$  is an amplitude modulation factor.

$\omega_0$  is the carrier frequency (in radians).

$\phi(t)$  is the phase modulation (in radians) about the carrier.

This equation can be represented in quadrature form as:

$$S(t) = X(t) \cos \omega_0 t - Y(t) \sin \omega_0 t \quad (2)$$

where:

$$X(t) = M(t) \cos \phi(t).$$

$$Y(t) = M(t) \sin \phi(t).$$

We can use the quadrature components to form a complex function,  $\mu(t)$ , which is defined as the complex envelope of the signal  $S(t)$ :

$$\mu(t) = X(t) + jY(t) = M(t)e^{j\phi(t)} \quad (3)$$

where:

$$M(t) = (X^2(t) + Y^2(t))^{\frac{1}{2}}.$$

$$\phi(t) = \tan^{-1} (Y(t)/X(t)).$$

Therefore, since

$$S(t) = \text{Re}\{\mu(t)e^{j\omega_0 t}\} \quad (4)$$

the complex envelope eliminates the necessity to manipulate the carrier frequency in calculations with the original signal.

Using the complex envelope to represent a signal yields several interesting properties. The first of these properties is the relationship between the energy (E) of the signal and the energy in the complex envelope. Energy is defined as:

$$\begin{aligned} E &= \int_{-\infty}^{\infty} |S(t)|^2 dt \\ &= \int_{-\infty}^{\infty} \frac{1}{2} M^2(t) + \frac{1}{2} (X^2(t) - Y^2(t) \cos 2\omega_0 t - X(t)Y(t) \sin 2\omega_0 t) dt \\ &= \int_{-\infty}^{\infty} \frac{1}{2} M^2(t) dt \\ 2E &= \int_{-\infty}^{\infty} M^2(t) dt. \end{aligned} \quad (5)$$

Now considering the energy in the complex envelope, we write:

$$\begin{aligned} \int_{-\infty}^{\infty} |\mu(t)|^2 dt &= \int_{-\infty}^{\infty} \mu(t) \mu^*(t) dt \\ &= \int_{-\infty}^{\infty} M^2(t) dt = 2E. \end{aligned} \quad (6)$$

Thus, the complex envelope contains twice the energy of the signal that it represents.

Another important property to consider is the convolution

of the complex envelopes of a signal and the impulse response of a filter. If we consider the frequency content of the complex envelope of a signal, we note:

$$\begin{aligned}
 S(\omega) &= \int_{-\infty}^{\infty} S(t) e^{-j\omega t} dt \\
 &= \frac{1}{2} \int_{-\infty}^{\infty} \{ \mu(t) e^{j\omega_0 t} + \mu^*(t) e^{-j\omega_0 t} \} e^{-j\omega t} dt \\
 &= \frac{1}{2} \{ M(\omega - \omega_0) + M^*(-\omega - \omega_0) \}.
 \end{aligned} \tag{7}$$

Therefore, we can represent the frequency content of the filter complex envelope,  $\lambda(t)$ , as:

$$H(\omega) = \frac{1}{2} \{ L(\omega - \omega_0) + L^*(-\omega - \omega_0) \}.$$

Now

$$\begin{aligned}
 h(t) &= \frac{1}{2} \{ \frac{1}{2} \pi e^{j\omega_0 t} \int_{-\infty}^{\infty} L(\Omega) e^{j\Omega t} d\Omega + \frac{1}{2} \pi e^{-j\omega_0 t} \int_{-\infty}^{\infty} L^*(\Omega) e^{-j\Omega t} d\Omega \} \\
 &= \frac{1}{2} \{ e^{j\omega_0 t} \lambda(t) + e^{-j\omega_0 t} \lambda^*(t) \} \\
 &= \text{Re} \{ \lambda(t) e^{j\omega_0 t} \}.
 \end{aligned} \tag{8}$$

Hence, given an input signal,  $S_i(t) = \text{Re} \{ \mu(t) e^{j\omega_0 t} \}$ , an output signal,  $S_o(t) = \text{Re} \{ \beta(t) e^{j\omega_0 t} \}$ , and the impulse response of a filter,  $h(t) = \text{Re} \{ \lambda(t) e^{j\omega_0 t} \}$ , we find that:

$$\begin{aligned}
 S_o(t) &= \int_{-\infty}^{\infty} h(\tau) S_i(t-\tau) d\tau = \frac{1}{2} \{ \beta(t) e^{j\omega_0 t} + \beta^*(t) e^{-j\omega_0 t} \} \tag{9} \\
 &= \frac{1}{4} \int_{-\infty}^{\infty} \lambda(\tau) \mu(t-\tau) e^{j\omega_0 t} d\tau + \frac{1}{4} \int_{-\infty}^{\infty} \lambda^*(\tau) \mu^*(t-\tau) e^{-j\omega_0 t} d\tau \\
 &\quad + \frac{1}{4} \int_{-\infty}^{\infty} \lambda(\tau) \mu^*(t-\tau) e^{-j\omega_0 (2\tau-t)} d\tau + \frac{1}{4} \int_{-\infty}^{\infty} \lambda^*(\tau) \mu(t-\tau) e^{j\omega_0 (t-2\tau)} d\tau.
 \end{aligned}$$

Since the third and fourth integrals equal zero,

$$S_o(t) = \text{Re}\{ \frac{1}{2} e^{j\omega_o t} \int_{-\infty}^{\infty} \lambda(\tau) \mu(t-\tau) d\tau \} \quad (10)$$

Therefore, we can equate values of  $S_o(t)$  (eqs 9 and 10) and find that:

$$\beta(t) = \frac{1}{2} \int_{-\infty}^{\infty} \lambda(\tau) \mu(t-\tau) d\tau = \frac{1}{2} \{ \lambda(t) * \mu(t) \} \quad (11)$$

Thus, the complex envelope of the output of the filter is half of the convolution of the complex envelopes of the input signal and the impulse response of the filter.

In addition to deterministic signals, we want to consider the use of the complex envelope with stationary random processes. If we consider a process,  $n(t)$ , that is band-limited between the frequencies of  $\pm\omega$  around a carrier frequency  $\Omega$ , we can represent  $n(t)$  as

$$n(t) = \text{Re}\{ \tilde{n}(t) e^{j\Omega t} \}.$$

We now consider the correlation function,  $R_n(\tau)$ , of the process.

$$\begin{aligned} R_n(\tau) &= E\{n(t) n^*(t-\tau)\} \\ &= \frac{1}{2} [ \frac{1}{2} \{ R_{\tilde{n}}(\tau) e^{j\Omega\tau} + R_{\tilde{n}}^*(\tau) e^{-j\Omega\tau} \} \\ &\quad + \frac{1}{2} \{ E(\tilde{n}(t) \tilde{n}(t-\tau)) e^{j(2\Omega t - \tau)} + E(\tilde{n}^*(t) \tilde{n}^*(t-\tau)) e^{-j(2\Omega t - \tau)} \} ] \end{aligned}$$

It can be shown that since  $n(t)$  is stationary,  $E(\tilde{n}(t) \tilde{n}(t-\tau))$  and  $E(\tilde{n}^*(t) \tilde{n}^*(t-\tau))$  are equal to zero. Hence,

$$R_n(\tau) = \frac{1}{2} \text{Re}\{R_{\tilde{n}}(\tau) e^{j\Omega\tau}\} \quad (12)$$

It follows that the spectral density of the process is

$$S_n(\omega) = \frac{1}{4} \{S_{\tilde{n}}(\omega - \Omega) + S_{\tilde{n}}^*(-\omega - \Omega)\} \quad (13)$$

We want to also consider what happens when we pass a stationary process through a wideband filter. If we let  $S_i(\omega)$  be the input spectral density,  $S_o(\omega)$  be the output spectral density, and  $H(j\omega)$  be the filter spectral density, we know that

$$S_o(\omega) = S_i(\omega) |H(j\omega)|^2 \quad (14)$$

It is possible to represent  $|H(j\omega)|^2$  as

$$|H(j\omega)|^2 = [h_{LP}(\omega - \Omega) + h_{LP}^*(-\omega - \Omega)] [h_{LP}^*(\omega - \Omega) + h_{LP}(-\omega - \Omega)]$$

We can then show that

$$S_o(\omega) = S_i(\omega) |h_{LP}(\omega)|^2 \quad (15)$$

and correspondingly that,

$$R_o(\tau) = \text{Re}\{R_{\tilde{o}}(\tau) e^{j\Omega\tau}\} \quad (16)$$

We also want to consider the case where the bandpass noise process is Gaussian. If we assume that the signals and noise have passed through a filter whose passband includes the signal spectrum, but is much wider than the signal spectrum, the noise can be represented by a complex envelope,



$n(t) = \text{Re}\{\tilde{n}(t)e^{j\Omega t}\}$ . It has been shown that with these assumptions,

$$R_{\tilde{n}}(\tau) = \frac{N_0}{2} \delta(\tau) \quad (17)$$

where

$N_0/2$  = amplitude of the noise spectral density

$\delta(\tau)$  = Dirac delta function.

#### Determination of the Matched Filter

In the infancy of radar, the signal-to-noise ratio was a popular measure of the effectiveness of a radar receiver for combating noise. The idea was to design the receiver to provide the maximum signal-to-noise ratio which made it easier to distinguish a signal from background noise. Signal-to-noise ratio, however, is only indirectly related to radar performance measures such as the probability of detecting a target, range and Doppler estimation accuracy, and so forth. It should be noted that, in many cases, decision theory leads to solutions identical to those obtained by maximizing the signal-to-noise ratio (Ref 1:143).

To begin, suppose we have an input waveform,  $V(t) = s_i(t) + n_i(t)$ , where:  $s_i(t)$  = input signal component  
 $n_i(t)$  = noise component,

and an output waveform from a filter,  $y(t) = s_o(t) + n_o(t)$ .

We want to determine the linear filter,  $h(t)$ , which maximizes the output signal-to-noise ratio at time  $t=t_m$ . The signal-

to-noise ratio is then defined as:

$$\frac{S}{N} = \frac{s_o^2(t_m)}{n_o^2(t_m)} \quad (18)$$

Assuming the noise is white and Gaussian, we know using Eq 17 that

$$R_n(\tau) = \delta(\tau) \frac{N_o}{2}$$

Hence, we can write:

$$\frac{S}{N} = \frac{\left| \int_{-\infty}^{t_m} s_i(\tau) h(t_m - \tau) d\tau \right|^2}{\frac{N_o}{2} \int_{-\infty}^{t_m} h^2(t_m - \tau) d\tau}$$

If we consider the signal-to-noise ratio in the frequency domain and apply the Schwarz inequality to the numerator, we see that:

$$\frac{S}{N} = \frac{1}{2\pi} \frac{\left| \int H(\omega) S_i(\omega) e^{j\omega t_m} d\omega \right|^2}{\frac{N_o}{2} \int |H(\omega)|^2 d\omega} \leq \frac{1}{2\pi} \frac{\int |S_i(\omega)|^2 d\omega}{\frac{N_o}{2}} \quad (19)$$

If  $H(\omega)$  is chosen so that equality exists, then

$$H(\omega) = \frac{2k}{N_o} S_i^*(\omega) e^{-j\omega t_m} \quad (20)$$

Thus, we see that the optimum filter in white Gaussian noise is

$$h(t) = \frac{2k}{N_o} s_i^*(t_m - t) \quad (21)$$

A similar derivation can be made using the complex envelope representation of the signals and the filter impulse response. We seek the linear filter,  $h(t)$ , that maximizes the output signal-to-noise ratio at some time,  $t_m$ . Let the input signal be  $x(t) = \text{Re}\{\tilde{x}(t)e^{j\omega_0 t}\}$ , the filter impulse response be  $h(t) = \text{Re}\{\tilde{h}(t)e^{j\omega_0 t}\}$ , and the output signal be  $y(t) = \text{Re}\{\tilde{y}(t)e^{j\omega_0 t}\}$ . We also assume that we have white Gaussian noise that has passed through a bandpass filter and that  $R_n(\tau) = \text{Re}\{R_{\tilde{n}}(\tau)e^{j\omega_0 \tau}\}$  and  $R_{\tilde{n}}(\tau) = \frac{N_0}{2}\delta(\tau)$ . We have already found that the optimum receiver in white Gaussian noise is  $K x^*(t_m - t)$ .

Hence we can say

$$\begin{aligned}\tilde{y}(t) &= \frac{1}{2} \int \tilde{x}^*(t_m - t + \tau) \tilde{x}(\tau) d\tau \\ &= R_{\tilde{x}}(t - t_m)\end{aligned}\quad (22)$$

Using our definition of the signal-to-noise ratio as before, and considering it in terms of frequency, we see that:

$$\frac{S}{N}(t=t_m) = \frac{\frac{1}{4} |\int \tilde{X}(f) \tilde{H}(f) e^{j2\pi f t_m} df|^2}{\frac{N_0}{4} \int |H(f)|^2 df} \quad (23)$$

Using the Schwarz inequality, we note that:

$$\frac{S}{N} \leq \frac{\int |X(f)|^2 df \int |H(f)|^2 df}{N_0 \int |H(f)|^2 df} \quad (24)$$

To get equality,  $\tilde{H}(f) = \tilde{X}^*(f) e^{-j\omega_0 t_m}$ , and hence,  $\tilde{h}(t) = \tilde{X}^*(t_m - t)$ .

### Uncertainty Relationship

In this section we want to develop parameters that characterize a signal and form the basis of comparisons between signals to accurately measure target parameters. We develop these parameters in terms of the complex envelopes of signals,  $S(t) = \text{Re}\{f(t)e^{j\Omega t}\}$ . We begin with a series of definitions:

$\bar{\omega}$  = mean frequency deviation

$$= \frac{\int \omega |F(\omega)|^2 d\omega}{\int |F(\omega)|^2 d\omega} = -j \frac{\int f^*(t) \left[ \frac{df(t)}{dt} \right] dt}{\int |f(t)|^2 dt} \quad (25)$$

$\overline{\omega^2}$  = mean square frequency deviation

$$= \frac{\int \omega^2 |F(\omega)|^2 d\omega}{\int |F(\omega)|^2 d\omega} = \frac{\int \left| \frac{df(t)}{dt} \right|^2 dt}{\int |f(t)|^2 dt} \quad (26)$$

$\Delta\omega$  = root mean square frequency deviation

$$\Delta\omega^2 = \overline{\omega^2} - (\bar{\omega})^2 = \overline{(\omega - \bar{\omega})^2} \quad (27)$$

$\bar{t}$  = mean duration deviation

$$= \frac{\int t |f(t)|^2 dt}{\int |f(t)|^2 dt} = j \frac{\int F^*(\omega) \left[ \frac{dF(\omega)}{d\omega} \right] d\omega}{\int |F(\omega)|^2 d\omega} \quad (28)$$

$\overline{t^2}$  = mean square duration

$$= \frac{\int t^2 |f(t)|^2 dt}{\int |f(t)|^2 dt} = \frac{\int \left| \frac{dF(\omega)}{d\omega} \right|^2 d\omega}{\int |F(\omega)|^2 d\omega} \quad (29)$$

$\Delta t$  = root mean square duration

$$\Delta t^2 = \overline{t^2} - (\overline{t})^2 = \overline{(t-\overline{t})^2} \quad (30)$$

$$\overline{\omega t} = -\frac{j}{2} - j \frac{\int t f^*(t) \left[ \frac{df(t)}{dt} \right] dt}{\int |f(t)|^2 dt} \quad (31)$$

Now the frequency modulation in the signal can be determined by calculating the quantity  $(\overline{\omega t} - \overline{\omega} \overline{t})$ .

If we choose  $\overline{\omega} = 0$ ,  $\overline{t} = 0$ , and  $\int |f(t)|^2 dt = 1 = 2E$ , then we can write:

$$\overline{\omega t} = -\frac{j}{2} - j \int t f^*(t) \left[ \frac{df(t)}{dt} \right] dt$$

$$|\overline{\omega t}|^2 = -\frac{1}{4} + \left| \int t f^*(t) \left[ \frac{df(t)}{dt} \right] dt \right|^2$$

Applying the Schwarz inequality we get:

$$|\overline{\omega t}|^2 + \frac{1}{4} \leq \overline{t^2} \cdot \overline{\omega^2}$$

Since  $\overline{\omega t}$  is a real number, we can define the uncertainty relation as

$$\overline{\omega^2} \cdot \overline{t^2} - (\overline{\omega t})^2 \geq \frac{1}{4} \quad (32)$$

The uncertainty relation can also be defined as

$$\Delta \omega \Delta t \geq \frac{1}{2} \quad (33)$$

Because of this relationship, we see that a decrease in the duration must result in an increase in bandwidth and vice-versa.

### Optimum Receiver in White Gaussian Noise

To determine the optimum receiver in the presence of white Gaussian noise, we consider the most simple case - signal known exactly. The Bayes detection strategy for an exactly known signal is a likelihood-ratio test. Knowledge of the form of the likelihood ratio permits specification of a test statistic which describes the structure of the optimum Bayes detection receiver (Ref 1:291).

To derive an expression for the likelihood ratio, it is convenient to employ the sampling-theorem waveform representation. The low-pass sampling theorem can be stated as follows:

Consider a signal,  $s(t)$ , bandlimited to  $(-\omega_c, \omega_c)$ . If  $s(t)$  is sampled at values spaced  $\frac{1}{2\omega_c}$  sec apart, then  $s(t)$  can be completely specified by those samples using the expression:

$$s(t) = \sum_{k=-\infty}^{\infty} s\left(\frac{k}{2f_c}\right) \frac{\sin 2\pi f_c \left(t - \frac{k}{2f_c}\right)}{2\pi f_c \left(t - \frac{k}{2f_c}\right)} \quad (34)$$

where  $f_c = \omega_c/2\pi$  (Ref 1:53).

It can also be shown that

$$\int_0^T s^2(t) dt \approx \frac{1}{2f_c} \sum_{k=1}^{2f_c T} f^2\left(\frac{k}{2f_c}\right) \quad (35)$$

where  $T$  is the signal duration (Ref 1:56).

Assuming the signal is known and bandlimited between  $(-f_c, f_c)$  and the noise is also bandlimited, we can write the

input waveform,  $\underline{V}$ , where  $\underline{V} = \underline{s} + \underline{n}$ , as  $V_i = s_i + n_i$  where  $V_i = V(\frac{i}{2f_c})$ ,  $s_i = s(\frac{i}{2f_c})$ , and  $n_i = n(\frac{i}{2f_c})$ ;  $i = 1, 2, \dots, 2f_c T$ .

The hypothesis  $H_1$  (signal plus noise) can be represented by the conditional joint probability density function

$$p(\underline{V}|H_1) = \frac{1}{(2\pi)^{N/2} (N_0 f_c)^N} \exp \left[ -\frac{1}{2f_c N_0} \sum_{i=1}^N (V_i - s_i)^2 \right] \quad (36)$$

where  $N = 2f_c T$  and the power spectral density of white noise is  $N_0/2$ . Similarly, the hypothesis  $H_0$  (noise alone) can be represented by the conditional joint probability density function

$$p(\underline{V}|H_0) = \frac{1}{(2\pi)^{N/2} (f_c N_0)^N} \exp \left( -\frac{1}{2f_c N_0} \sum_{i=1}^N V_i^2 \right) \quad (37)$$

The likelihood-ratio expression is given by

$$\Lambda(\underline{V}) = \frac{p(\underline{V}|H_1)}{p(\underline{V}|H_0)} = \frac{\exp \left\{ -\frac{1}{2f_c N_0} \sum_{i=1}^N (V_i - s_i)^2 \right\}}{\exp \left\{ -\frac{1}{2f_c N_0} \sum_{i=1}^N V_i^2 \right\}} \quad (38)$$

Using Eq 35, we can write Eq 38 as

$$\Lambda(\underline{V}) = \ell(V(t)) = \exp \left\{ -\frac{1}{N_0} \int_0^T s^2(t) dt + \frac{2}{N_0} \int_0^T V(t) s(t) dt \right\} \quad (39)$$

Now

$$\ln \ell(V(t)) = -\frac{E}{N_0} + \frac{2}{N_0} \int_0^T V(t) s(t) dt \quad (40)$$

Thus, the likelihood ratio receiver is

$$\frac{2}{N_0} \int_0^T V(t) s(t) dt \underset{H_0}{\overset{H_1}{>}} \text{Threshold} + \frac{E}{N_0} \quad (41)$$

The test statistic (quantity on the left side of the inequality) represents the cross correlation of  $V(t)$  and  $s(t)$ . The matched filter is used to realize the cross correlator (Ref 1:292-294). We note that the matched filter is also the optimum detector since it comes from the likelihood ratio test using the Neyman-Pearson criterion; that is, for a fixed probability of false alarm ( $P_{FA} = \alpha$ ), the matched filter is the receiver that maximizes the probability of detection.

#### Ambiguity Function

When the information about the radar environment is limited, it appears reasonable to study the performance of matched filter receivers since they provide the optimum performance in white Gaussian noise. We now want to determine the resolution capabilities (ability to recognize a particular target in the presence of others) of a radar waveform assuming our radar uses a matched filter receiver (Ref 5:116).

Suppose we have two point targets differing in range. Let  $x(t) = a(t) \cos(\omega_0 t + \phi(t))$  be the return from the first target and  $x(t-\tau) = a(t-\tau) \cos(\omega_0 (t-\tau) + \phi(t-\tau))$  be the return from the second target, where the difference in range between the signals is  $\text{Range} = \frac{C\tau}{2}$ , where  $C$  is the speed of light.

From our discussion of the complex envelope representation



of signals, we know that

$$x(t) = \text{Re} \{ \mu(t) e^{j\omega_0 t} \}$$

$$x(t-\tau) = \text{Re} \{ \mu(t-\tau) e^{j\omega_0 t} e^{-j\omega_0 \tau} \}$$

To resolve these targets we seek to maximize the integral square error,  $\overline{\epsilon^2}$ , between them. We define that error as:

$$\overline{\epsilon^2} = \int_{-\infty}^{\infty} |x(t) - x(t-\tau)|^2 dt \quad (42)$$

In complex envelope form, we find:

$$\begin{aligned} \overline{\epsilon^2} &= \int [\mu(t) - \mu(t-\tau) e^{-j\omega_0 \tau}] [\mu^*(t) - \mu^*(t-\tau) e^{j\omega_0 \tau}] dt \\ &= 2 \int |\mu(t)|^2 dt - 2 \text{Re} \{ e^{-j\omega_0 \tau} \int \mu^*(t) \mu(t-\tau) dt \} \\ &= 2[2E] - 2 \text{Re} \{ C(\tau) e^{-j\omega_0 \tau} \} \end{aligned} \quad (43)$$

where  $C(\tau) = \int \mu^*(t) \mu(t-\tau) dt$ . Hence, we want to minimize  $C(\tau)$  to get the best resolution in range.

We now want to define the delay resolution constant, which is:

$$T_R = \frac{\int |C(\tau)|^2 d\tau}{|C(0)|^2} = \frac{\int |M(f)|^4 df}{[\int |M(f)|^2 df]^2} \quad (44)$$

where  $M(f)$  is the Fourier transform of  $\mu(t)$ . We then define the range resolution constant as  $\Delta R = \frac{CT_R}{2}$ . We also define the effective bandwidth as  $B_e = \frac{1}{2T_R}$ . Thus, we see that to improve range resolution (i.e., make  $\Delta R$  less), we must

increase the effective bandwidth.

Since we now have a method to determine range resolution, we use the time-frequency duality principle to formulate a method to determine velocity resolution. We now consider two point targets at the same range, but having different radial velocities. Again trying to maximize the integral square error between the signals, we find

$$\overline{\epsilon^2} = \int |x(t) - x(t)e^{j\omega_D t}|^2 dt \quad (45)$$

where  $\omega_D$  is the difference in the target Doppler-shift frequencies (in radians).

Then

$$\overline{\epsilon^2} = 2 \int |x(t)|^2 dt - 2 \operatorname{Re} \{ \int x(t) x^*(t) e^{j\omega_D t} dt \} \quad (46)$$

We want to minimize  $\int x(t) x^*(t) e^{j\omega_D t} dt$ , and we note that  $\int x(t) x^*(t) e^{j2\pi V t} dt = \int X^*(V) X(V+W) dW = K_X(V)$  where

$$\omega_D = 2\pi V \text{ and } X(V) = \text{Fourier Transform of } x(t) \quad (47)$$

Therefore, we want the spectral correlation function  $K_X(V)$  to be as impulsive as possible to achieve good resolution. We now define the frequency resolution constant as

$$F_r = \frac{\int |u(t)|^4 dt}{[\int |u(t)|^2 dt]^2} \quad (48)$$

where  $X(t) = \operatorname{Re}\{u(t)e^{j\omega_0 t}\}$ .

We also define the effective time duration,  $\tau_e = \frac{1}{F_r}$ . Thus,

for two targets at the same range, they can be resolved in velocity only if

$$\frac{\Delta\omega_p}{2} \geq F_r$$

where  $\Delta\omega_p$  = difference in target Doppler shift.

Another way of considering target resolution for a single parameter is that targets must occupy different bins. A bin is the interval whose width corresponds to the half-power width of the central lobe of the matched filter response. When we consider this approach, we assume that clutter in other bins is low and that there is no coupling between the two parameters. When we considered range resolution, we had

$$C(\tau) = \int \mu^*(t) \mu(t-\tau) dt = \int |M(f)|^2 e^{j2\pi f\tau} df$$

To define the bin width, we say

$$|C(\tau_0)| = \frac{1}{2} = \left| \int |M(f)|^2 e^{j2\pi f\tau} df \right| \quad (49)$$

If we consider the Taylor series approximation of the magnitude squared, we get

$$C(\tau)^2 = \frac{1}{2} = |C(0)|^2 + \frac{1}{2} \frac{\partial^2 |C(\tau)|^2}{\partial \tau^2} \bigg|_{\tau=0} \tau^2 \quad (50)$$

Assuming  $C(0)=1$ , then  $\frac{1}{2} = 1 + \frac{1}{2} (-2\beta^2) \tau^2$  where  $\beta$  is the rms bandwidth. Note that  $\beta^2 = \overline{\omega^2}$  as defined in Eq 26. Then  $\beta^2 \tau^2 = \frac{1}{2}$  and  $\tau = \frac{1}{\sqrt{2}\beta}$ . We must consider the two-sided nature

of the response, and thus,

$$\tau' = 2\tau = \sqrt{2}/\beta \quad (51)$$

We then define the range bin as  $r_b = \frac{\tau' C}{2} = \frac{C}{\sqrt{2}\beta}$ . For velocity resolution, we found  $|K_u(v)|^2 = |\int |u(t)|^2 e^{j2\pi vt} dt|^2$ . Doing the Taylor Series approximation about  $v=0$ , we find

$$|K_u(v)|^2 = \frac{1}{2} = |K_u(0)|^2 + \frac{1}{2} \frac{\partial^2 |K_u(v)|^2}{\partial v^2} \bigg|_{v=0} \cdot v^2 \quad (52)$$

Assuming normalized signal energy,  $K_u(0) = 1$ , then we find  $v_b = \frac{\sqrt{2}}{t_d}$  where  $t_d$  is the rms duration. Note that  $t_d^2 = \bar{t}^2$  as defined in Eq 29. Therefore, the Doppler bin is defined as

$$\dot{r}_b = \frac{\lambda v_b}{2} = \frac{\lambda}{\sqrt{2} t_d} \quad (53)$$

For the more general case, we now consider two point targets that differ in both range and velocity, and consider the resolution capabilities of a waveform. Again we want to maximize the integral square error between the signals, and note that

$$\begin{aligned} \overline{\epsilon^2} &= \int |X(t)e^{j\omega_D t} - X(t-\tau)|^2 dt \\ &= 2 \int |X(t)|^2 dt - \text{Re}\{\int X(t)X^*(t-\tau)e^{j\omega_D t} dt\} \end{aligned} \quad (54)$$

We then want to minimize the value  $\int X(t)X^*(t-\tau)e^{j\omega_D t} dt$ . If we then consider the complex envelope representation of this value, we get

$$x(\tau, \omega_D) = \int \mu(t) \mu^*(t-\tau) e^{j\omega_D t} dt \quad (55)$$

which we will define as the ambiguity function of the waveform. Thus, the ambiguity function describes the complex envelope of the signal at the output of the matched-filter receiver as a function of radar target range and radial velocity. This function, which is determined solely by the transmitted waveform and matched-filter characteristics, has proved to be extremely useful in the design and study of radar signals, since it answers questions about resolution, ambiguities, measurement precision, and clutter rejection.

The ambiguity surface, or the plot of  $|x(\tau, \omega)|^2$  above the  $\tau, \omega$  plane, plays a central part in the analysis of resolution. The width of the central peak of the ambiguity surface is a measure for close target resolution in delay and Doppler, and subsidiary spikes and low-level parts of the surface give an indication of the problem of self-clutter and target masking by mutual interference (Ref 5:114).

#### Properties of the Ambiguity Function

The ambiguity function (Eq 55) in the frequency domain is given as:

$$x(\tau, \omega) = \int M(f+\omega) M^*(f) e^{-j2\pi f\tau} df \quad (56)$$

The maximum value of the ambiguity function occurs at the origin:

$$|x(\tau, \omega)| \leq x(0, 0) = 2E$$

Often, the signal is normalized, and the maximum value is unity.

Now  $x(-\tau, -\omega) = e^{j\omega\tau} x^*(\tau, \omega)$ . Since  $|x^*(\tau, \omega)| = |x(\tau, \omega)|$ , we have  $|x(-\tau, -\omega)| = |x(\tau, \omega)|$ . This means that the ambiguity function is symmetric with respect to the origin.

We then consider the volume of the square magnitude of the ambiguity function:

$$\text{Volume} = \iint |x(\tau, \omega)|^2 d\tau d\omega$$

$$\text{Now } F_{\tau, V} |x(\tau, V)|^2 = \iint |x(\tau, V)|^2 e^{-j2\pi\delta\tau} e^{-j2\pi\mu V} d\tau dV = |x(\delta, \mu)|^2$$

If we let  $\delta=0$ ,  $\mu=0$ , then

$$\iint |x(\tau, V)|^2 d\tau dV = |x(0, 0)|^2 = (2E)^2$$

This means that waveform design can never get rid of the volume under  $|x(\tau, V)|^2$ ; all proper waveform design can do is to try to move the inevitable volume to regions in the  $\tau, V$  plane that are relatively free of clutter and other interfering targets (Ref 12:3-12).

Targets at ranges and velocities such that  $|x(\tau, V)|$  is about as large as  $|x(0, 0)|$  are indistinguishable to the radar. The width of the peak about  $|x(0, 0)|$  defines the resolution of the waveform. Other peaks away from  $|x(0, 0)|$  correspond to ambiguities of the waveform.

Waveforms having  $x(\tau, V)=0$  in the region of the  $\tau, V$  plane where clutter exists generally have good clutter-rejec-

tion properties. More precisely, the summation over the entire  $\tau, V$  plane of the product of  $|x(\tau, V)|^2$  with the clutter determines the total interfering clutter signal (Ref 12:3-10).

### Digital Signal Processing

In computing the ambiguity function of waveforms, a linear convolution of samples of the complex envelope representation of the Doppler-shifted signal and the time-reversed, conjugate of the signal is performed. This procedure results in a cut of the ambiguity function for a particular Doppler frequency. The linear convolution itself is done by taking the Fourier transform of the two arguments, multiplying the transforms, and then performing the inverse Fourier transform of the product. The Fourier transforms of the samples are computed using a fast Fourier transform procedure. We shall now briefly describe the basic theory required to justify this approach.

To begin, we must consider how to sample the two arguments in order to accurately reproduce the arguments. This can be determined by the uniform sampling theorem for lowpass signals which may be stated as: Theorem. If a signal  $x(t)$  contains no frequency components for frequencies above  $f = W$  Hz, then it is completely described by instantaneous sample values uniformly spaced in time with period  $T_s < \frac{1}{2W}$ . The signal can be exactly reconstructed from the sampled waveform by passing it through an ideal lowpass filter with bandwidth

B, where  $W < B < f_s - W$  with  $f_s = \frac{1}{T_s}$ . The frequency  $2W$  is referred to as the Nyquist frequency (Ref 16:68). Thus, we know that we must sample the arguments at intervals less than or equal to the Nyquist rate ( $\frac{1}{2W}$ ) in order to accurately reproduce the waveform. These samples of the arguments form finite-duration sequences.

For a finite duration sequence, we define the discrete Fourier transform,  $X(k)$ , of that sequence,  $x(n)$ , as:

$$X(k) = \sum_{n=0}^{N-1} x(n) w_N^{kn}, \quad 0 \leq k \leq N-1 \quad (57)$$

0, otherwise

where

$N$  is the number of samples in the sequence

$$w_N = e^{-j(2\pi/N)}.$$

The inverse of the discrete Fourier transform is given by:

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) w_N^{-kn}, \quad 0 \leq n \leq N-1 \quad (58)$$

0, otherwise (Ref 4:100)

Highly efficient algorithms are available to compute the discrete Fourier transform (DFT), known as Fast Fourier transforms, of a finite-duration sequence. For this reason, it is computationally efficient to consider implementing a convolution of two sequences by computing their discrete Fourier transforms, multiplying, and computing the inverse discrete Fourier transform. Now consider two  $N$ -point



sequences,  $x_1(n)$  and  $x_2(n)$ , and denote their linear convolution as:

$$x_3(n) = \sum_{m=0}^{N-1} x_1(m) x_2(n-m) \quad (59)$$

It is straightforward to verify that  $x_3(n)$  is of length  $2N-1$ . If it is obtained by multiplying the discrete Fourier transforms of  $x_1(n)$  and  $x_2(n)$ , then each of these discrete Fourier transforms,  $X_1(k)$  and  $X_2(k)$ , must also have been computed on the basis of  $2N-1$  points. The easiest way to consider expanding the  $N$ -point sequences to  $2N-1$  point sequences is that  $N-1$  zeros are added to the end of each of the sequences. Thus if we define

$$\begin{aligned} X_1(k) &= \sum_{n=0}^{2N-2} x_1(n) w_{2N-1}^{nk} \\ X_2(k) &= \sum_{n=0}^{2N-2} x_2(n) w_{2N-1}^{nk} \\ x_3(n) &= \frac{1}{2N-1} \sum_{k=0}^{2N-2} X_1(k) X_2(k) w_{2N-1}^{-nk} \end{aligned}$$

then  $x_3(n)$  will be the linear convolution of  $x_1(n)$  and  $x_2(n)$ . Of course, a linear convolution would also be achieved if the discrete Fourier transforms were computed on the basis of more than  $2N-1$  points, but would not in general be achieved if the discrete Fourier transforms were computed on the basis of fewer points (Ref 4:110-111).

There are several fast Fourier transform methods avail-

able to compute the discrete Fourier transform of a sequence; however the one we shall consider is called the decimation-in-time method. In this method, the discrete Fourier transform is decomposed into successively smaller DFT computations. Thus, the original DFT, as represented by Eq 57, is separated into its even and odd points such that

$$X(k) = \sum_{n \text{ even}} x(n) W_N^{nk} + \sum_{n \text{ odd}} x(n) W_N^{nk} \quad (60)$$

Using the fact that  $W_N^2 = W_{N/2}$ , we can write the DFT as

$$\begin{aligned} X(k) &= \sum_{n=0}^{(N/2)-1} x(2n) W_{N/2}^{rk} + W_N^k \sum_{n=0}^{(N/2)-1} x(2n+1) W_{N/2}^{nk} \\ &= G(k) + W_N^k H(k) \end{aligned} \quad (61)$$

Then  $G(k)$  and  $H(k)$  would be broken down in the same manner until only two-point transforms remain. Thus, the number of complex multiplications and additions needed to compute the DFT would be reduced from  $N^2$  to  $N \log_2 N$ . (Ref 4:290-294).

#### Clutter Models and Applications

The function of the clutter model is to represent the random clutter process in such a manner that system performance can be determined. Ideally the clutter model accurately reflects the clutter process in a realistic manner. Practically, the clutter data base is rarely adequate to achieve this objective. The art of modeling the clutter involves a

compromise between accurate representation and analytic convenience (Ref 9:37).

Clutter is characterized by its mean or median equivalent backscattering cross-section, amplitude probability density function, and its power spectral density or equivalent autocorrelation function. Angel clutter, radar echoes primarily caused by birds and insects, has velocities that can vary from 15 to 60 knots depending on wind conditions. Ground clutter has a narrow intrinsic spectral width determined by internal clutter motion induced by the wind. The ground clutter spectral density is usually assumed to have a Gaussian shape. Sea clutter power spectral density is usually assumed to be Gaussian shaped with an rms velocity spread depending upon sea state (a measure of the height of sea waves). Sea clutter has a mean velocity dependent upon wind speed, radar polarization, and direction of the sea. Weather clutter has a mean velocity proportional to the wind speed. Wind turbulence and wind shear effect (caused by the change in wind speed with height) can cause considerable spectral spreading (Ref 9:5-17).

The spectrum of echoes from rain, snow, and chaff may be deduced from the motion of the wind since these particles are good wind tracers. For the radar frequencies which we will consider, we can convert scatterer motion to clutter spectra through the Doppler equation. The spectrum of weather clutter is found to be dominated by two effects which describe

the motion of the wind-driven particles: turbulence and wind shear. Turbulence describes the fluctuations of the wind about its mean motion. The center of the spectrum will be at the frequency corresponding to the mean wind speed with respect to the radar beam center. Experiments indicate that the width of the turbulence spectrum, expressed by its standard deviation,  $\sigma_{\text{turb}}$ , is approximately  $\sigma_{\text{turb}} \approx 1.0$  m/sec.

The wind shear effect is due to the change in wind speed with height, which can usually be represented as a linearly increasing function over the elevation extent of the beam. The spectrum width due to wind shear may be evaluated for the linear wind shear model by assuming a Gaussian antenna pattern. As a result,

$$\sigma_{\text{shear}} = 0.42 k R \theta_2 \quad (62)$$

where:

$k$  is a constant representing the velocity gradient component vs height in the direction of the beam (m/sec/km).

$R$  is the slant range to the clutter (km).

$\theta_2$  is the two-way, half-power antenna beamwidth in elevation (radians).

Although this equation seems to indicate that the spectrum width can increase indefinitely, in practice the value of  $\sigma_{\text{shear}}$  is limited to about 6 m/sec. For arbitrary radar azimuth, an average value of  $k$  is 4.0 m/sec/km. The total spectrum width,  $\sigma_v$ , due to both turbulence and wind shear, is:

$$\sigma_v = (\sigma_{\text{turb}}^2 + \sigma_{\text{shear}}^2)^{1/2} \text{ (m/sec)} \quad (63)$$

Because of the wind shear effect, the center of the clutter spectrum has an offset value corresponding to the wind speed at the beam center (Ref 2:387).

Data from a number of investigations at various frequencies show that the spectrum of sea clutter in velocity units is a nearly linear function of wind speed, or alternatively, sea state, as plotted in the figure below. The width of the Doppler spectrum may be obtained from this figure simply through the Doppler equation. The literature indicates that the spectrum width is not very sensitive to pulse width, azimuth, depression angle, or polarization (Ref 2:388).

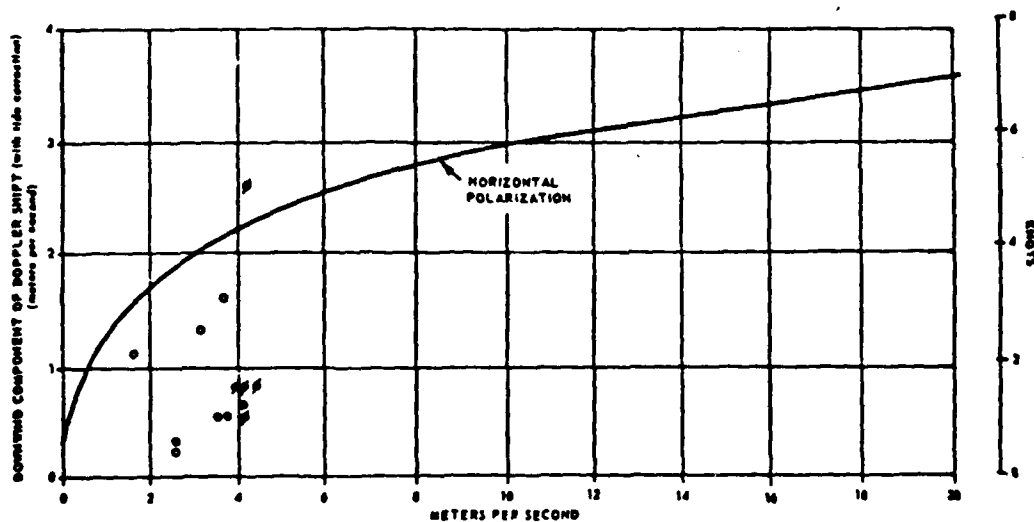


Figure 1. Spectrum of Sea Clutter (in velocity)  
(Ref 2:388)

Land clutter extends from the radar site out to the radar horizon. The mean velocity of the land clutter is obviously zero. The standard deviation of the velocity spectrum of land clutter is mainly a function of the surface wind speed. As an example, the standard deviation of the velocity spectrum for clutter echoes from a wooded area with a 20 knot surface wind is  $\sigma_v = 0.25$  (m/sec) (Ref 2:390).

In all of the examples given, the spectral width has been expressed in m/sec and must be multiplied by  $2/\lambda$  to convert to Doppler frequency. The relationship between the physical motion of the clutter scatterers and the spectral width has been found to conform to the Doppler equation throughout most of the microwave region of interest (Ref 2:390).

When considering how clutter is distributed in range, we can, in general, assume that it is uniformly distributed in the specified clutter areas. Therefore, we only need to determine the starting and ending ranges for each of the clutter areas. The uniform probability density function is given by:

$$p(\tau) = \frac{c}{2(b-a)} \quad (64)$$

where

b is the ending range of the clutter area.

a is the starting range of the clutter area.

c is the speed of light.

If we now desire to determine a model of the clutter

in both delay and Doppler, we want a probability density function as a function of both variables. In this paper the variables are assumed to be independent, therefore, we can determine the joint probability function as

$$p(\tau, v) = \frac{c}{2(b-a)} \left\{ \frac{1}{2\sigma_v^2} \exp \left[ \frac{-(v-\bar{v})^2}{(2\sigma_v^2)} \right] \right\} \quad (65)$$

where:

$a, b, c$  are as defined previously.

$\bar{v}$  is the mean Doppler frequency of the clutter.

$\sigma_v$  is the standard deviation of the Doppler frequency.

Since the clutter power often is far in excess of that from the target, it is necessary to choose the transmit waveform on the basis of the clutter distributions. The Ambiguity Diagram, a plot of the magnitude of the ambiguity function as a function of delay and Doppler, has been used as a tool for evaluating the choice of waveform in specific environments. It has proven quite successful for suggesting specific waveforms for resolving multiple targets in range or range rate and in certain highly specific clutter environments. In the extreme case, all signal waveforms are equally good (or bad) as long as they are not compared against a specific radar environment. Fortunately, there are inevitably some bounds on the location, velocity, and intensity of clutter (Ref 2: 389).

Assuming that we can now model the clutter for the environment, we want to use the model to assist in selecting the radar waveform. If we assume the transmitted signal to be

$$S(t) = A\mu(t)e^{j\omega_0 t}$$

the returns from the scatterers will be

$$S_c(t) = \sum_{n=1}^N A_n \mu(t-\tau_n) e^{j(\omega_0 - \Omega_n)(t-\tau_n)} \quad (66)$$

where

$A_n$  is the amplitude of the  $n$ th return.

$N$  is the number of scatterers.

$\tau_n$  is the delay of scatterers from matched filter.

$\Omega_n$  is the Doppler of scatterers from matched filter.

Assuming a matched-filter receiver, the total output response will be:

$$\begin{aligned} y(t) &= \frac{1}{2} \sum_{m=1}^N \sum_{n=1}^N A_n \int [\mu(\gamma - \tau_n) e^{j(\omega_0 - \Omega_n)(\gamma - \tau_n)}] \\ &\quad [\mu^*(\tau_0 - t + \gamma - \tau_m) e^{-j(\omega_0 - \Omega_m)(T_0 - t + \gamma - \tau_m)}] d\gamma \\ &= \frac{1}{2} \sum_{m=1}^N \sum_{n=1}^N A_n e^{-j(\omega_0 - \Omega_m)\tau_n} x(\tau_n, \phi_n) \end{aligned} \quad (67)$$

Now we consider the term  $e^{-j(\omega_0 - \Omega_m)\tau_n}$ . If the scatterers are modeled as randomly distributed in range, then  $\theta_n = (\omega_0 - \Omega_m)\tau_n$  is a random angle. Assuming independence, then  $\theta_n$  is modeled



by a uniform probability density function (pdf), and we get:

$$y(t) = \frac{1}{2} \sum_{n=1}^N A_n e^{-j\theta_n} x(\tau_n, \phi_n) \quad (68)$$

One measure of performance is the peak signal energy to clutter power ratio:

$$E\left\{\frac{S}{C}\right\} = \frac{\frac{A_T^2}{4} |x(0,0)|^2}{E\left\{\frac{1}{2} \left| \sum_{n=1}^N A_n e^{-j\theta_n} x(\tau_n, \phi_n) \right|^2\right\}} \quad (69)$$

By appropriate manipulation, we find:

$$E\left\{\frac{S}{C}\right\} = \frac{A_T^2 |x(0,0)|^2}{N \sum_{n=1}^N A_n^2 |x(\tau_n, \phi_n)|^2} \quad (70)$$

When the clutter is dense, it is useful to replace the discrete scatterers by a continuous density function in delay and Doppler space. Thus, we can write:

$$E\left\{\frac{S}{C}\right\} = \frac{\sigma_T |x(0,0)|^2}{N \bar{\sigma} \iint f(\tau, \omega) |x(\tau, \omega)|^2 d\tau d\omega} \quad (71)$$

where

$f(\tau, \omega)$  is the clutter pdf.

$\sigma_T$  is the target radar-cross section.

$N$  is the total number of scatterers.

$\bar{\sigma}$  is the average radar-cross section ( $E\{A_n^2\} = \bar{\sigma}$ ).

Thus, with an appropriate probability density function

modeling the clutter and the ambiguity function, the signal-to-clutter ratio can be estimated (Ref 12:3-39).

In order to approximate the average radar-cross section of clutter returns, a brief discussion of clutter amplitude models will be given. The Gaussian or Rayleigh envelope clutter model is applicable to distributed clutter that is returned from a spatially continuous distribution of scatterers with no subset of scatterers predominating. This type of clutter is usually associated with weather clutter, chaff, sea clutter observed with a low resolution (pulse width  $> 0.5 \mu\text{sec}$ ) or with a high-resolution radar at high grazing angles ( $\phi > 5^\circ$ ), and land clutter observed from high grazing angles over undeveloped terrain. The clutter cross-sectional area is proportional to the power envelope resulting in an exponential probability density function for the clutter cross-section (C):

$$f_p(C) = \frac{1}{\bar{C}} \exp [-C/\bar{C}]$$

where  $\bar{C}$  is the mean clutter cross-section and is given by

$$\bar{C} = \bar{\sigma}^0 C' \quad (72)$$

where  $\bar{\sigma}^0$  is the mean backscattering coefficient.

$C'$  is the area intercepted by the radar cell (Ref 9:38-39).

The log-normal clutter model has been used to model high-resolution sea-clutter data, where the sea clutter is

observed at grazing angles less than 5 degrees. In addition, ground clutter observed at low grazing angles has also been modeled using the log-normal model. The mean value of the clutter cross-section is given by:

$$\bar{C} = \rho_C C_m \quad (73)$$

where

$C_m$  is the median value of the clutter cross-section.

$\rho_C$  is the mean-to-median ratio, where

$$\rho_C = \exp (\sigma_p^2/2)$$

where

$\sigma_p$  = twice the standard deviation of the underlying normal distribution.

The distribution function for the cross-sectional area is given by

$$F_p(C) = \frac{1}{2} [1 + \operatorname{erf} (\frac{1}{\sqrt{2}\sigma_p} \ln \frac{C}{C_m})] \quad (74)$$

The mean or median cross-sectional area is determined from a knowledge of the clutter's backscattering coefficient. Table I gives some reported values of  $\sigma_p$  for various types of clutter (Ref 9:45-47). Other useful clutter models can be found in Scheleher.

Table I. Log-Normal Clutter Parameters  
(Ref 9:47)

Terrain/Sea State	Frequency	$\phi$	$\sigma_p$
Sea State 2-3	X	$4.7^\circ$	1.382
Sea State 3	$K_u$	$1-5^\circ$	1.440-.960
Sea State 4	X	$0.24^\circ$	1.548
Sea State 5*	$K_u$	$0.50^\circ$	1.634
Land Clutter (Discrete)	S	low	3.916
Land Clutter (Distributed)	S	low	1.380
Land Clutter*	P- $K_a$	$10^\circ-70^\circ$	.728-2.584
Rain Clutter	X and 95 GHz	-	.680
Radar Angels	-	-	1.352-1.620
*Extrapolated Values			
$\phi$ is the grazing angle			

### III. Software Developed to Resolve the Problem

#### Approach

The method of waveform selection discussed in this paper requires three primary products associated with software development:

(1) a method to calculate the magnitude of a Doppler cut of the ambiguity function for typical radar waveforms.

(2) a method to produce a three-dimensional display of the ambiguity function magnitude as a function of delay and Doppler.

(3) a method to calculate the signal-to-clutter ratio for a specified target, waveform, and clutter environment. All the software developed to perform these requirements was done by the author with the exception of the fast Fourier transform program and the general three-dimensional graphics program. The software developed by the author was programmed using the Fortran V language on the Data General Eclipse computer.

In this section of the paper, the algorithms developed by the author will be explained. Each of the three products mentioned above will be discussed in detail. Appendix B contains a system flowchart for the software. Appendix D contains the program listings of the software developed by the author.

### Calculating the Magnitude of the Ambiguity Function

As we have stated before, the ambiguity function describes the complex envelope of the signal at the output of the radar receiver as a function of radar target range and radial velocity (Ref 12:3-3). It was defined as:

$$x(\tau, \omega) = \int \mu(t) \mu^*(t-\tau) e^{j\omega t} dt$$

The ambiguity function (which is a correlation function) can also be written as a convolution:

$$x(\tau, \omega) = \mu(\tau) e^{j\omega\tau} \mu^*(-\tau) \quad (75)$$

If we defined the arguments of the convolution as finite sequences, we could calculate the ambiguity function by performing a linear convolution. To get the arguments into finite sequence representation, we must appropriately sample the complex envelope representation to form an N-point sequence, and then add at least N-1, zero-valued samples to the end of the sequence to get it into a form to perform a linear convolution. The linear convolution itself can be implemented by calculating the discrete Fourier transform of the sequences, multiplying, and calculating the inverse discrete Fourier transform of their product. The discrete Fourier transform and its inverse can be quickly calculated using the decimation-in-time method of the fast Fourier transform.

To appropriately sample the waveform to produce the N-point sequences, the Nyquist rate of the waveform must be determined. The time-bandwidth product, signal duration multiplied by the signal bandwidth, of the complex envelope representation provides the Nyquist rate (Ref 3:254). If the waveform is sampled at intervals of  $1/\text{time-bandwidth}$  or less, then N-point finite sequences are developed. By adding at least N-1 zero-valued samples to these sequences, the linear convolution can be performed.

The Eclipse computer has a library program which calculates the fast Fourier transform and its inverse using the decimation-in-time method in which the transform is decomposed into successively smaller increments. The sequences which will be transformed must have their total number of samples equal to some power of two, up to a maximum of 1024, for the program to run correctly (Ref 17:1-1,3-4). Because of this requirement, we must sample the waveform at intervals of  $1/N$  or less, where N is the smallest value of two raised to a power which is greater than the time-bandwidth product. Also, we must add N zero-valued samples to the sequence to have the proper form for linear convolution. If we want good resolution of the ambiguity function, we may have to increase the number of samples of the waveform to get the desired results.

As an example, assume we want to calculate the ambiguity function of a rectangular pulse which has a pulse width of 1 sec and an amplitude of 1. The time-bandwidth product of the

complex envelope would be 1, and therefore, two samples of the waveform should allow us to calculate the ambiguity function. Let us assume that to get the resolution desired for the ambiguity function, we make eight samples of the waveform. The samples would be made at 1/8 sec intervals, starting at 1/8 sec. We would also have add eight zero-valued samples to the sequences to perform the linear convolution. Hence, the finite sequence representing  $\mu(\tau)e^{j\omega\tau}$  would be:

$$e^{j\omega(1/8)}, e^{j\omega(1/4)}, e^{j\omega(3/8)}, e^{j\omega(1/2)}, e^{j\omega(5/8)}, \\ e^{j\omega(3/4)}, e^{j\omega(7/8)}, e^{j\omega}, 0, 0, 0, 0, 0, 0, 0, 0.$$

The finite sequence representing  $\mu^*(-\tau)$  would be:

$$0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1.$$

Before ending this discussion of forming appropriate sequences, we must consider how to sample waveforms with repeated pulses or with a phase-shift coded pulse. The problem to be considered here is to equally sample each element of the waveform to eliminate any possible bias. Suppose, for example, that the waveform has two pulses, each 1 sec in duration, and a repetition time of 3 secs. If we choose to make 16 samples of the waveform, then the samples will be separated by intervals of  $(3+3)/16$  sec or  $3/8$  sec. Then there will be two samples made of the first pulse and three samples made of the second pulse. If the calculations are



done now, the linear convolution and ambiguity function results would be incorrect. To correct this situation, the samples are "weighted" to insure each elemental part of the waveform is sampled properly and equally. In our example, we know the duration of the waveform and the number of samples we wish to make. By truncating the value of the expression, (number of samples/waveform duration) x (pulse width) to the integer value, we now have the correct number of samples to make of each pulse. In this example, the waveform will be specified by the first twelve samples; thus, we simply zero-fill the remaining four samples and continue the process as discussed previously.

After forming the appropriate finite sequences for  $\mu(\tau)e^{j\omega\tau}$  and  $\mu^*(-\tau)$ , the linear convolution must be performed. The library program to compute the fast Fourier transform of both sequences is called. Due to an error in this program, the results must be conjugated to provide the correct results. This program also requires the results to be divided by the total number of samples (2N) to be completely accurate. The transformed sequences are then multiplied together on a sample-by-sample basis. The resulting product is then inverse transformed using the library program. The results must again be conjugated and divided by the number of samples to get the correct ambiguity function values. These results are samples in delay of a Doppler cut of the ambiguity function. The magnitude of these samples is then calculated, and we have

the desired results.

In the discussion so far, we have not considered how to handle waveforms with Gaussian-shaped pulses nor those having some form of frequency modulation. In order to normalize the magnitude of the ambiguity function when Gaussian-shaped pulses are used, we must determine a weighting factor. This weighting factor was determined on the basis that the mean of a Gaussian pulse was  $\frac{1}{2}$ (pulse width) and the standard deviation was  $1/6$ (pulse width). Using  $\pm 3$  standard deviations from the mean, insures that our approximation is within 2% of the actual result. Thus we want to determine the weighting factor, A such that  $A \int s^2(t) dt = 1$ . Now

$$s(t) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \{-(t-\bar{t})^2/2\sigma^2\} \quad (76)$$

where

$$\sigma = \frac{1}{6} \text{ (pulse width).}$$

$$\bar{t} = \frac{1}{2} \text{ (pulse width).}$$

Thus we want

$$A \int_0^P \frac{1}{2\pi\sigma^2} \exp \{-(t-\bar{t})^2/\sigma^2\} dt = 1 \quad (77)$$

Substituting  $x = (t-\bar{t})/\sigma$  into Eq 77, we get:

$$\frac{A}{2\pi\sigma} \int_{-\bar{t}/\sigma}^{(PW-\bar{t})/\sigma} \exp(-x^2) dx = 1$$

$$A \int_{-3}^3 \exp(-x^2) dx = \frac{\pi(PW)}{3}$$

Using a standard normal distribution table, we can determine the value of  $\int_{-3}^3 \exp(-x^2) dx = 0.9974\sqrt{\pi}$ . Therefore,

$$A = [\pi(PW)]/[3(0.9974)\sqrt{\pi}] = 0.59236 (PW) \quad (78)$$

Thus, to normalize the magnitude of the ambiguity function samples, simply multiply each sample by the weighting factor A.

When the waveform has been frequency modulated, the effects on the ambiguity function are significant. We recall that the complex envelope representation,  $\mu(t)$  was also defined as  $\mu(t) = |\mu(t)|e^{j\phi(t)}$ , where  $\phi(t)$  will contain the frequency modulation. In this study only linear frequency modulation (FM) and V-FM techniques were considered. For linear FM, the complex envelope has the form:  $\mu(t) = |\mu(t)|e^{j\pi kt^2}$ . The instantaneous frequency of the complex envelope is  $f_i = \frac{1}{2\pi} \frac{d\phi(t)}{dt} = kt$  with  $k$  determining the slope of the FM. If the instantaneous frequency is swept over the band,  $B$ , during the signal duration,  $T$ , the absolute value of  $k$  is given as  $|k| = B/T$ . The sign of  $k$  determines the direction of the sweep, with  $k$  positive for a frequency upswing and negative for a downswing (Ref 5:169-170).

Meanwhile, if V-FM is chosen, the waveform is split into two parts, one for the downswing and the other for the upswing. The complex envelope for V-FM has the form:

$$\begin{aligned} \mu(t) = & [u(t) - u(t - PW/2)] |\mu(t)| e^{-j\pi kt^2} \\ & + [u(t - PW/2) - u(t - PW)] |\mu(t)| e^{j\pi kt^2} \end{aligned} \quad (79)$$

where  $u(t)$  is the unit step function. Again, assuming the instantaneous frequency is swept over the band  $B$  during the signal duration  $T$ , then the absolute value of  $k$  is given as  $|k| = B/T$  (Ref 5:176).

As an example, suppose we again choose our waveform to have a pulse width of 1 sec, an amplitude of 1, and been sampled every  $1/8$  sec (i.e. 8 samples are made of the waveform). Suppose we also want to FM the waveform and sweep over a band of 25 Hz. If we choose linear FM, the sequence representing  $\mu(\tau)e^{j\omega\tau}$  will be:

$$\begin{aligned} & e^{j(\omega/8 + 25\pi/64)}, e^{j(\omega/4 + 25\pi(4)/64)}, \dots, e^{j(\omega + 25\pi)}, \\ & 0, 0, 0, 0, 0, 0, 0, 0. \end{aligned}$$

The sequence representing  $\mu^*(-\tau)$  will be:

$$\begin{aligned} & 0, 0, 0, 0, 0, 0, 0, 0, e^{-j(25\pi/64)}, e^{-j(25\pi(4)/64)}, \dots, \\ & e^{-j(25\pi)}. \end{aligned}$$

When a waveform has phase-shift coding, the amplitude of the complex envelope is modulated. For example, suppose we choose our waveform to have a pulse width of 1 sec and an amplitude of 1, and to have 8 samples made of it. If we assume it is phase-shift coded as a 5-element Barker code,

the phase for each element would be  $0, 0, 0, \pi, 0$  (i.e. the amplitude of the waveform for each element would be  $1, 1, 1, -1, 1$ ).

Then the sequence representing  $\mu(\tau)e^{j\omega\tau}$  will be:

$$e^{j\omega(1/5)}, e^{j\omega(2/5)}, e^{j\omega(3/5)}, -e^{j\omega(4/5)}, e^{j\omega},$$

$$0, 0, 0, 0, 0, 0, 0, 0, 0, 0$$

since we also had to weight our sampling of the waveform.

The sequence representing  $\mu^*(-\tau)$  will be:  $0, 0, 0, 0, 0, 0, 0, 0, 0, 0,$   
 $0, 0, 1, 1, 1, -1, 1.$

Thus, we have described the general principles used to calculate the magnitude of the ambiguity function for the waveforms considered. The software developed to calculate the ambiguity function magnitudes implements these principles. Magnitudes calculated using this software were consistently within one percent of the actual results as long as adequate sampling was made of the waveform.

#### Producing Three-Dimensional Plots of the Magnitude

One of the most useful tools in selecting the waveform is the plot of the ambiguity function magnitude. We have already discussed the method used to calculate the magnitude of samples made in delay for a Doppler cut of the ambiguity function. A method is needed to select and properly format these magnitude samples (made for various Doppler cuts of the ambiguity function) for use by the three-dimensional graphics program. This graphics program, called PLTTRNS,

was written by R.W. Schafer, July 3, 1978, at the Georgia Institute of Technology. The PLTTRNS program requires the input to be a file of a two-dimensional array of complex numbers in binary format. The array is assumed square and of minimum size  $64 \times 64$  and of maximum size  $512 \times 512$ . The program provides three possible display modes on the Textronics storage scope: contour, perspective, and single-line. The contour plot and three-dimensional perspective plot can only display an array of data of size  $65 \times 65$ . If the input array is larger than  $64 \times 64$ , an appropriate number of rows and columns are decimated prior to display. Therefore, the data we want displayed must be of size  $64 \times 64$ .

The problem we now face is how to select the data we want displayed and how to properly format those data for the PLTTRNS program. Since our two-dimensional array contains 64 delay samples for each of 64 different Doppler cuts of the ambiguity function, we must decide which delay samples and Doppler frequencies to choose. The software developed in this study allows the user to choose which delay sample to begin the display and the option to skip samples. For example, if 128 delay samples are available over the delay times of -1 sec to 1 sec, the user could choose to start at sample 64 (corresponding to time 0 sec) and observe every sample out to sample 127 (corresponding to time  $63/64$  sec). The user could also choose to observe every other sample, starting at sample 1 and ending at sample 127, to have a display covering the

entire delay interval. This choice is simply a matter of the resolution desired by the user.

Although the choice of the Doppler frequencies is again a matter of the resolution desired by the user, greater care should be taken in the selection process. Prior knowledge of the resolution required for similar waveforms can assist in this selection. With no prior knowledge, the user should begin by observing closely spaced (interval of 1 Hz) cuts and then increasing the spacing until the resolution meets the user's desires. As with the delay samples, the user chooses the starting value of the Doppler cuts. The user also specifies the interval between the Doppler cuts, and the 64 cuts are then calculated as explained previously.

After the magnitudes of the samples of the ambiguity function cut have been calculated, the magnitudes are then quantized to a value between 0 and 200. The quantization occurs because the PLTTRNS program must have the data in integer form. The 200 quantization levels provide the resolution necessary for smoothing of the curves of the three-dimensional graphs. After quantizing the magnitude samples, they are transformed into complex data with the imaginary part set to zero, and then stored into a two-dimensional complex array. When all 64 Doppler cuts have been calculated and stored in the complex array, the data are written onto a file called "Data". The PLTTRNS program is then called, and the three-dimensional plot of the magnitude, as well as a

contour plot, can then be made.

#### Calculating the Signal-to-Clutter Ratio

As stated previously, one of the constraints in selecting a waveform is how well the waveform is able to detect desirable targets in the environment. The signal-to-clutter ratio is the value which will be used to indicate the performance of the waveform in the environment. As derived previously, the signal-to-clutter ratio is given by Eq 71. In the theoretical discussion of this paper, guidance was given on how to establish the probability density function of clutter returns as a function of delay and Doppler and on how to establish the average radar cross-section of the clutter returns. The user must establish the radar cross section of the desired target and estimate the number of scatterers reflecting energy. Since the magnitudes of the ambiguity function are always normalized in this study,  $|x(0,0)|^2 = 1$ .

The significant task left for the software is to calculate the volume of the product of the magnitude squared of the ambiguity function and the clutter probability density function. In this procedure the boundaries of the clutter areas are defined in terms of delay and Doppler, as well as the boundaries of the ambiguity function. The interval between the Doppler cuts of the ambiguity function must be determined and specified. It is important to note that no limit exists in this software for the number of Doppler cuts



that can be made. The interval should be chosen to insure that all significant changes in the ambiguity function magnitudes are considered. The necessary resolution in delay should also dictate the number of samples to be made of the waveform. The three-dimensional plots of the ambiguity function magnitude can greatly assist in these parameter selections.

After the necessary parameters are entered, the software begins its work. The algorithm begins by calculating the magnitudes of the samples for the first two Doppler cuts and storing those data in an array. Then the first two magnitude samples of both Doppler cuts are considered. The value of the clutter probability density function is then calculated for each of the four points. If the user wishes to input his own clutter distribution, he will be prompted to do so if any of the points is included in a clutter area. The product of the magnitude squared and the clutter probability density function is then calculated for each of the four points. The average value of the four products is then calculated, and the minimum and maximum products are identified. These three products are each then multiplied by the incremental area of the  $\tau, \omega$  plane to produce an incremental value of the minimum, maximum, and average volume. The incremental area of the  $\tau, \omega$  plane is simply the product of the interval between the samples in delay and the interval between the Doppler cuts. Also, the incremental volumes are added to a cumulative sum for the respective minimum, maximum, and average volumes.

After this process is completed, the second and third magnitude samples for both Doppler cuts are considered. Again, a product of the magnitude squared and the value of the clutter probability density function for each of these four points is calculated. Incremental volumes for the minimum, maximum, and average products are calculated and added to the respective cumulative sums.

This process of considering the next magnitude samples for the first two Doppler cuts is continued until all the samples of those cuts are considered. The final incremental values will be those for the last and next-to-last samples. Then we simply consider the second and third Doppler cuts and repeat the entire process again. The entire procedure ends with the last and next-to-last Doppler cuts repeating the process.

As an elementary example, suppose we have three magnitude samples for each of three Doppler cuts of an ambiguity function. Suppose the Doppler cuts are made at frequencies of 1,0, and -1 Hz and the magnitude samples occur at delay values of 1,0, and -1 sec. Also, suppose the magnitude of the samples and the values of the probability density function for the specified points are given in the following table:

Table II. Example Values for Calculating the Signal-to-Clutter Ratio

Doppler (Hz)	Delay (sec)	Magnitude of Amb Fcn	Value of Clutter PDF
-1	-1	0.1	0.1
-1	0	0.2	0.1
-1	1	0.1	0.1
0	-1	0.2	0.1
0	0	1.0	0.2
0	1	0.2	0.1
1	-1	0.1	0.1
1	0	0.2	0.1
1	-1	0.1	0.1

Considering the first two Doppler cuts (-1 and 0 Hz) and the first two delay samples (-1 and 0 sec) for each, we find the products of the magnitude squared and clutter pdf to be: 0.001, 0.004, 0.004, and 0.200. The average value would be 0.05225, the minimum would be 0.001, and the maximum would be 0.200. The incremental volumes would be the same since the incremental area is 1. Considering the next samples in delay (0 and 1 sec), we find the incremental volumes to be 0.001, 0.200, and 0.05225 for the minimum, maximum, and average volumes, respectively. Then we consider the next Doppler cuts (0 and 1 Hz) and find the incremental volumes for the first two delay samples (-1 and 0 sec) to be: 0.001, 0.200, and 0.05225 again. Similarly, we find the same incremental volumes for the last two delay samples (0 and 1 sec). The cumulative volumes would then be 0.004, 0.800,

and 0.209 for the minimum, maximum, and average volumes, respectively.

After the volumes have been calculated, a comparison can be made to determine if the volumes have a wide spread. As long as the spread between the minimum and maximum volumes is less than 20 percent of the maximum volume, we can assume the average volume to be an adequate approximation of the true volume. If the spread is greater than 20 percent, the program should be run again with smaller intervals between the samples and the Doppler cuts.

Once we have an acceptable approximation of the volume of the magnitude squared of the ambiguity function and the clutter probability density function, we then make the calculation of the signal-to-clutter ratio. The cumulative average volume is the value used in this calculation. Several signal-to-clutter ratios may need to be calculated for differing targets and types of clutter.

We note that a software program was also developed which generates a three dimensional plot of the probability density function of the clutter model as a function of delay and Doppler. This plot, used with a corresponding plot of the magnitude of the ambiguity function, can give the user a rough approximation of the signal-to-clutter ratio that exists for the particular waveforms being considered.

#### IV. Applications of Software in Waveform Selection

##### General Approach

As stated previously, the purpose of this study was to develop a method of selecting a waveform given the desired radar coverage and operational environment. Software programs have been developed in this study to provide products to assist the user in the selection process. In this section of the study, an explanation will be given of how to use these software products in selecting a waveform. This section begins with a discussion of how to analyze the given requirements of radar coverage and the environment. The next topic discussed is a method of determining the capabilities of a waveform, given the coverage requirements and environment. A discussion of how to make tradeoffs between the desired requirements and actual capabilities available is then given. The section concludes with the analysis of an example problem.

##### Analyzing Requirements

In this section we will briefly explain how to take a standard set of radar coverage requirements and environmental location and determine what factors to use with the software products available. We assume the standard requirements to be given as the unambiguous range, range resolution, acceptable blind speeds, and velocity resolution of desired targets. The general type of location, in which the radar is to be

operating, will also be given.

The unambiguous range, which is the range between range bins, can be used to determine the minimum delay required between range bins by the equation:

$$\Delta\tau = \frac{UR}{2C} \quad (80)$$

where:

UR is the unambiguous range.

$\Delta\tau$  is the delay between delay bins.

C is the velocity of light.

The range resolution requirement specifies the maximum range bin width and can be used to determine the minimum effective signal bandwidth by the equation:

$$\beta = \frac{C}{\sqrt{2}(RBW)} \quad (81)$$

where:

$\beta$  is the minimum effective signal bandwidth (see Eq 51).

RBW is the range bin width.

The blind speed requirement, which is the velocity between Doppler bins, can be used to determine the Doppler interval required between bins by the equation:

$$\Delta f = \frac{2V}{\lambda} \quad (82)$$

where:

$\Delta f$  is the Doppler interval.

V is the blind speed velocity.

$\lambda$  is the wavelength.

The velocity resolution requirement specifies the maximum Doppler bin width and can be used to determine the minimum effective signal duration by the equation:

$$t_d = \frac{\lambda}{\sqrt{2} \text{VBW}} \quad (83)$$

where:

$t_d$  is the minimum effective signal duration (see Eq 53).

$\lambda$  is the wavelength.

VBW is the velocity bin width.

With the minimum effective signal bandwidth and minimum effective signal duration, the minimum effective time-bandwidth product of the signal can be determined.

Since the general type of environment is also given, the types of associated clutter can be deduced. With an idea of the clutter, an analysis can be performed, as outlined in the theoretical discussion, to estimate the average radar cross-section of the clutter and to develop a probability density function of the clutter as a function of delay and Doppler. These data will be required to calculate the signal-to-clutter ratio.

#### Determining Waveform Capabilities

By analyzing the given requirements, the user can begin to use the software products in the selection process. At this point in the process, the user knows the minimum time-bandwidth product, the minimum spacing between delay bins

and between Doppler bins, and the maximum delay bin width and Doppler bin width. Also, a minimum signal-to-clutter ratio value of 30, which is an arbitrary estimate, must be achieved to detect a target.

Radar waveforms can be classified into groups according to the general properties of their ambiguity function magnitudes. We can assume the waveforms fall into one of four categories: constant-carrier pulse, irregular or noiselike pulse (e.g., phase-shift coded pulse), linear frequency modulated pulse, and repetitive pulses (Ref 6:1079).

The basic radar waveform is the constant-carrier pulse, which has a time-bandwidth product of unity. The magnitude of the ambiguity function, thus, is concentrated in a unity size cell in delay-Doppler space. Depending on the exact pulse shape, there may be sidelobes in delay and Doppler, but these will contain only an insignificant part of the total energy response. This type of waveform will be designated as a Class A waveform (Ref 6:1079).

The irregular or noiselike pulse, like a phase-shift coded pulse, has an ambiguity function magnitude resembling an inverted thumbtack. The dimensions of the central spike of the thumbtack function are  $(1/\text{bandwidth}) \times (1/\text{signal duration})$ , which define the size of the resolution cell. This type of waveform trades a small resolution cell against a potentially severe self-clutter problem. The resolution performance will be poor in a dense-target environment or when



a small target is to be detected in the presence of much stronger reflectors. This waveform type will be designated as a Class B1 waveform (Ref 6:1079).

When the instantaneous frequency or the carrier frequency is linearly varied or stepped, the waveform is classified as a linear FM signal. The magnitude of the ambiguity function of this signal can be viewed as a sheared version of the ambiguity function magnitude of the constant-carrier pulse. It follows that the size of the resolution cell must also be unity, and that little sidelobe energy need be outside the main response ridge. This implies an excellent resolution potential when targets, in delay-Doppler space, do not fall along the ridge, but essentially no resolution performance for targets that do fall along the ridge. The amount of shearing of the ridge is determined by the bandwidth swept by the FM; hence, the larger the bandwidth swept, the greater the shearing of the ridge. Signals of this type are designated as Class B2 waveform (Ref 6:1080).

If the waveform is a periodic repetition of a pulse whose bandwidth is large compared to the repetition frequency, the signal is called a repetitive pulse signal and is designated as a Class C waveform. The magnitude of the ambiguity function consists of a regular array of spikes in the delay-Doppler domain, with the delay spacing of the spikes equal to the repetition period and the Doppler spacing equal to the

repetition frequency. The dimensions of the ambiguous spikes are  $(1/\text{bandwidth}) \times (1/\text{pulse width})$  as in the case of the thumbtack function. However, instead of the uniform sidelobe pedestal, the response energy is concentrated in isolated spikes. This waveform type offers excellent resolution performance in a dense-target environment at the cost of a possible ambiguity problem (Ref 6:1080). The following table summarizes the characteristics of the magnitude of the ambiguity functions for the waveform classifications.

Table III. Waveform Classification and Ambiguity Functions

	Class A	Class B1	Class B2	Class C
Time-bandwidth product	unity	large compared with unity		
Ambiguity function	ridge	thumbtack	sheared ridge	bed-of-nails
Resolution cell size	unity	1/TB	unity	1/TB
Ambiguities	no	no	range-Doppler coupling	spikes
Sidelobes	low	high	low	low

(Ref 6:1080)

A three-dimensional diagram and a contour diagram of each waveform classification is contained in Appendix C. These will clearly demonstrate the characteristics which we have discussed.

With the information about the waveform requirements which is now available, the user should be able to eliminate certain waveform classifications from further consideration. Other waveform types should also be recognized as the most likely candidates to be selected. Because high-performance radars are very expensive, it is important that the user choose the optimum waveform in the sense that it will do the job in the least complicated manner. Thus, the user will have a prioritized list of the remaining waveform candidates to analyze further.

#### Applying Software Products

With the information now available, the user can further analyze the waveform by using the software products developed in this study. The user should begin this analysis phase with the least complicated waveform candidate. Since the analysis of the delay and Doppler bin widths and bin separations were only rough approximations in the preceding sections, the user will want to obtain a three-dimensional diagram and a contour plot (indicating the half-power points) of the magnitude of the ambiguity function. With this information, the user can more precisely establish the resolution capabilities and ambiguities associated with the waveform. If these resolution capabilities and ambiguities do not meet the initial requirements, the user should attempt to modify the waveform parameters until the requirements are met or

until the user concludes that, in fact, the waveform can not meet the requirements.

Assuming that the waveform now meets the resolution and ambiguity requirements, the user must insure that desired targets can be detected in the modeled clutter environment. To determine this, the user should use the software to calculate the signal-to-clutter ratio. If the signal-to-clutter ratio is 30 or above, then the user may assume the target will be detected. If the result is less than 30, the user should eliminate the waveform from further consider.

This process of establishing the resolution capabilities, ambiguities, and signal-to-clutter ratio should be continued for all candidate waveforms. If more than one waveform fulfills the requirements, the user may want to consider tradeoffs between signal complexity to gain improved waveform performance. If none of the candidate waveforms can fulfill the original requirements, the user must make tradeoffs in the requirements to obtain a waveform which would be the most acceptable.

#### Tradeoffs

For the case of several waveforms fulfilling the requirements, the user would normally use the least complex waveform. If a more complex waveform provides significantly improved performance, the user might accept the more complex waveform, especially if the cost to implement the more complex

waveform is within budget constraints. The user must determine whether the improved performance is great enough to outweigh the increased cost.

On the other hand, if none of the candidate waveforms fulfill the specified requirements, the user must determine which requirements can be changed and by how much. Perhaps the least flexible requirement is the signal-to-clutter ratio required to detect desired targets. The tradeoffs in resolutions and ambiguities must be determined by the user. By prioritizing the requirements, the user can revise the requirements until a waveform is found that fulfills them. If the waveform can be implemented within the cost constraints, it should then be selected. If the implementation will cost too much, the requirements must continue to be revised. Thus, the user's judgement is the most important factor in determining tradeoffs.

#### Example Problem

To make the preceding discussion of waveform selection more clear, an example problem of waveform selection will now be presented. We shall assume the following requirements have been given to us by the contractor:

Minimum unambiguous range - 25 km

Range resolution - .5 km

Minimum first blind speed - 200 km/hr = 55.6 m/s

Velocity resolution - 20 km/hr = 5.6 m/s

Operating environment - coastal radar  
 Minimum desired target size -  $4\text{m}^2$   
 Cost constraints are inflexible.  
 Operating frequency of radar - 3 GHz

We begin with a cursory analysis to determine the minimum time-bandwidth product needed by the waveform. The minimum effective bandwidth is first estimated by Eq 81:

$$\beta = \frac{3 \times 10^8 \text{ m/s}}{\sqrt{2} (.5 \times 10^3 \text{ m})} = 4.24 \times 10^5 \text{ Hz}$$

The minimum effective signal duration is estimated by Eq 83:

$$t_d = \frac{C}{\sqrt{2} f(\text{VBW})} = \frac{3 \times 10^8 \text{ m/s}}{\sqrt{2} (3 \times 10^9 \text{ Hz}) (55.6 \text{ m/s})}$$

$$= 1.27 \times 10^{-3} \text{ s}$$

Hence, the minimum effective time-bandwidth product must be approximately 538. Therefore, we know that the simple pulse, Class A, can not be used.

We now can approximate the delay and Doppler bin widths. The delay bin width is approximated as:

$$\text{Bin width} = \frac{2(\text{Range resolution})}{C} = \frac{2(500\text{m})}{3 \times 10^8 \text{ m/s}} = 3.33 \text{ } \mu\text{sec}$$

The Doppler bin width is approximated as:

$$\begin{aligned}\text{Doppler bin width} &= \frac{2(\text{Velocity Resolution})}{\lambda} \\ &= \frac{2(55.6 \text{ m/s})(3 \times 10^9 \text{ Hz})}{3 \times 10^8 \text{ m/s}} = 1.11 \text{ k Hz}\end{aligned}$$

The delay bins must be separated by approximately (using Eq 80):

$$\Delta \tau = \frac{25 \times 10^3 \text{ m}}{2(3 \times 10^8 \text{ m/s})} = 4.17 \times 10^{-5} \text{ sec} = 41.7 \text{ } \mu\text{sec}$$

The Doppler bins must be separated by approximately (using Eq 82):

$$\Delta f = \frac{2(556 \text{ m/s})(3 \times 10^9 \text{ Hz})}{3 \times 10^8 \text{ m/s}} = 11.1 \text{ k Hz}$$

Now since we normally would use phase-shift coding as the Class B1 waveform, we would find it impossible to achieve the required time-bandwidth product using a Class B1 waveform. More sophisticated noise-like coding could produce the required time-bandwidth product, but the cost of implementing it would be prohibitive. Therefore, we want to consider the two remaining waveform types, linear FM and repeated pulses.

If repetitive pulses are used, the pulse repetition time must be at least 41.7  $\mu\text{sec}$  to achieve the spacing in delay. This would correspond to a pulse repetition frequency of at most 24 kHz which would exceed the Doppler spacing requirement of 11.1 kHz. If we selected a PRT = 50  $\mu\text{sec}$ , a

pulse width of 5  $\mu$ sec, and a train of 10 pulses, we would achieve the desired time-bandwidth product.

If linear FM is used, the bandwidth to be swept could be 25 MHz and the pulse width 20  $\mu$ sec. This would provide the required time-bandwidth product. The resolution properties can be met except along the delay-Doppler coupled ridge. We can investigate the V-FM waveform which will reduce the coupling with a small increase in complexity. Therefore, we will now run the programs to produce the three-dimensional, ambiguity function magnitude diagrams and contour plots for the repetitive pulse, linear FM, and V-FM waveforms.

First we consider the repetitive pulse waveform. As shown in Figure 2, a plot of the magnitude of the central portion of the ambiguity function, the Doppler spacing between the 3-dB areas is approximately 17.9 KHz. Observing Figures 3 and 4, diagrams of the peak portion of the ambiguity function magnitude and 3-dB contour of magnitude squared, respectively, the Doppler resolution is approximately 995 Hz, and the delay resolution is approximately 2.0  $\mu$ sec. Figure 5 indicates that the delay spacing between the 3-dB areas is approximately 45.0  $\mu$ sec. As shown in Figure 6, the small subsidiary peaks between the main Doppler areas are at most 9dB below the main regions. Hence, the effects of ambiguities from these-subsidiary levels is negligible. Thus, the repetitive pulse waveform exceeds all resolution and ambiguity requirements.



Next we consider the linear FM waveform. As shown in Figures 7 and 8 (a plot of the ambiguity function magnitude and a plot of the -3dB and -6dB contours of the magnitude squared, respectively) the delay resolution is approximately 0.234  $\mu$ sec while the Doppler resolution is approximately 31 kHz. The delay resolution is well within limits; however, the Doppler resolution is poorer than required. Ambiguities exist only along the delay-Doppler ridge; however, it will be virtually impossible to resolve targets which correspond to points along the ridge. These may be acceptable to the user, especially if the signal-to-clutter ratio is high; therefore we will continue to consider this waveform.

As shown in Figures 9 and 10 (a plot of the ambiguity function magnitude and a plot of the -6dB contour of the magnitude squared, respectively), the V-FM waveform provides better delay and Doppler resolution than the linear FM waveform. The delay and Doppler resolutions for V-FM at the -6dB contour correspond to those for linear FM at the -3dB contour. With V-FM, however, two ridges of delay-Doppler ambiguities exist compared to the one ridge for linear FM. In any case, we will continue to consider the V-FM waveform as a candidate waveform.

We next must develop a clutter model for the operational environment. By modeling the environment, we can then determine the signal-to-clutter ratio for the candidate waveforms.

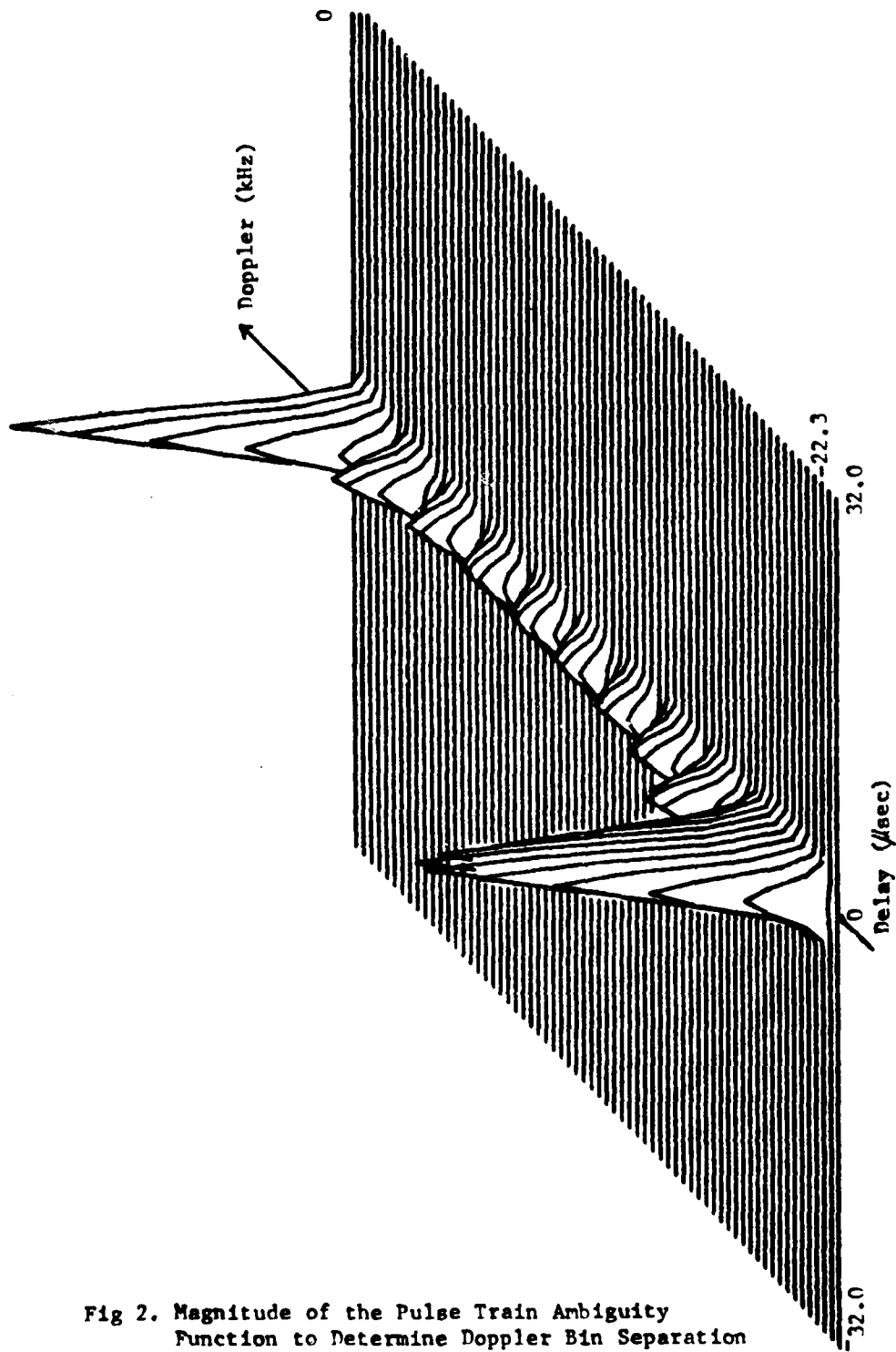


Fig 2. Magnitude of the Pulse Train Ambiguity Function to Determine Doppler Bin Separation

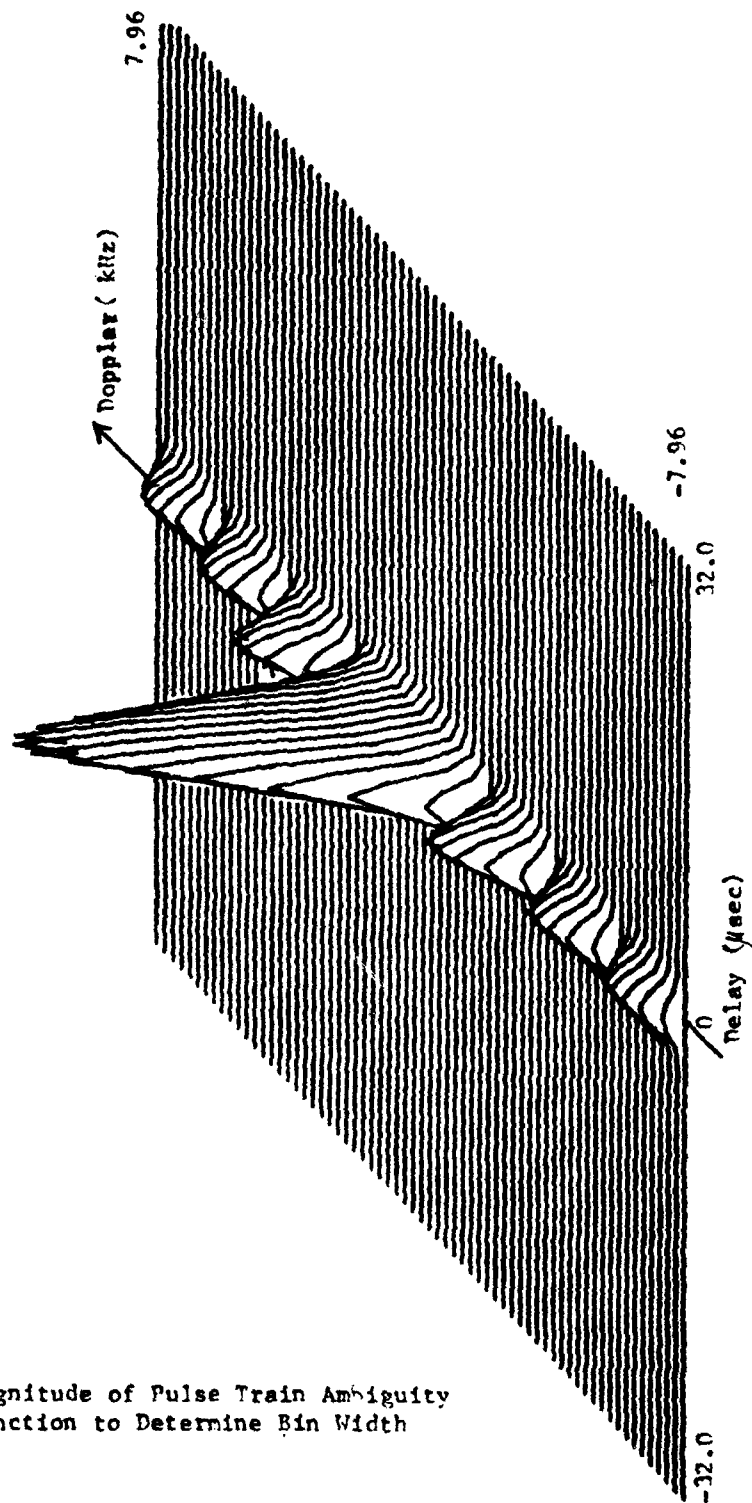


Fig 3. Magnitude of Pulse Train Ambiguity Function to Determine Bin Width

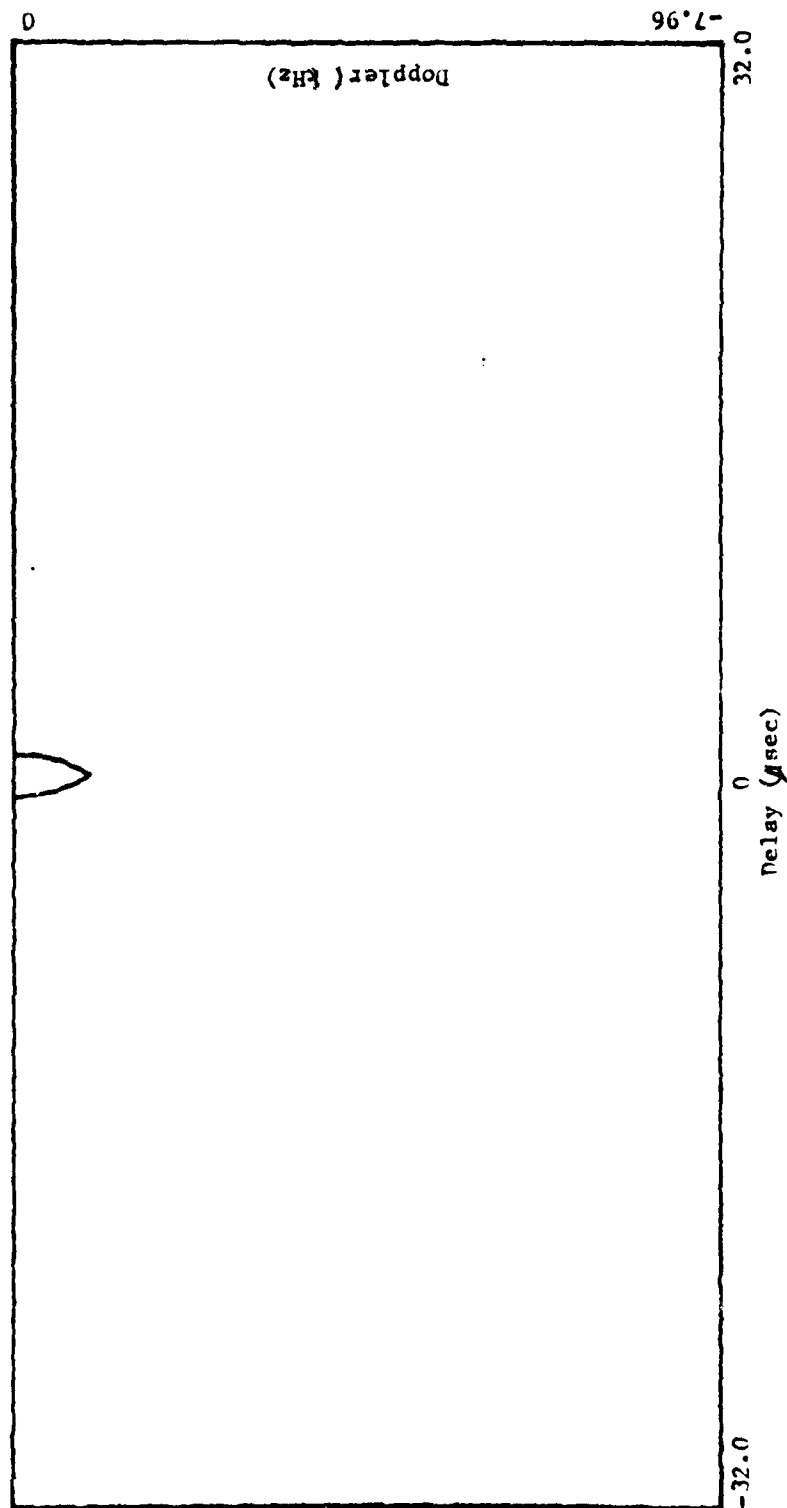


Fig 4. -3dB Contour of Magnitude Squared of the Pulse Train Ambiguity Function to Determine Bin Width

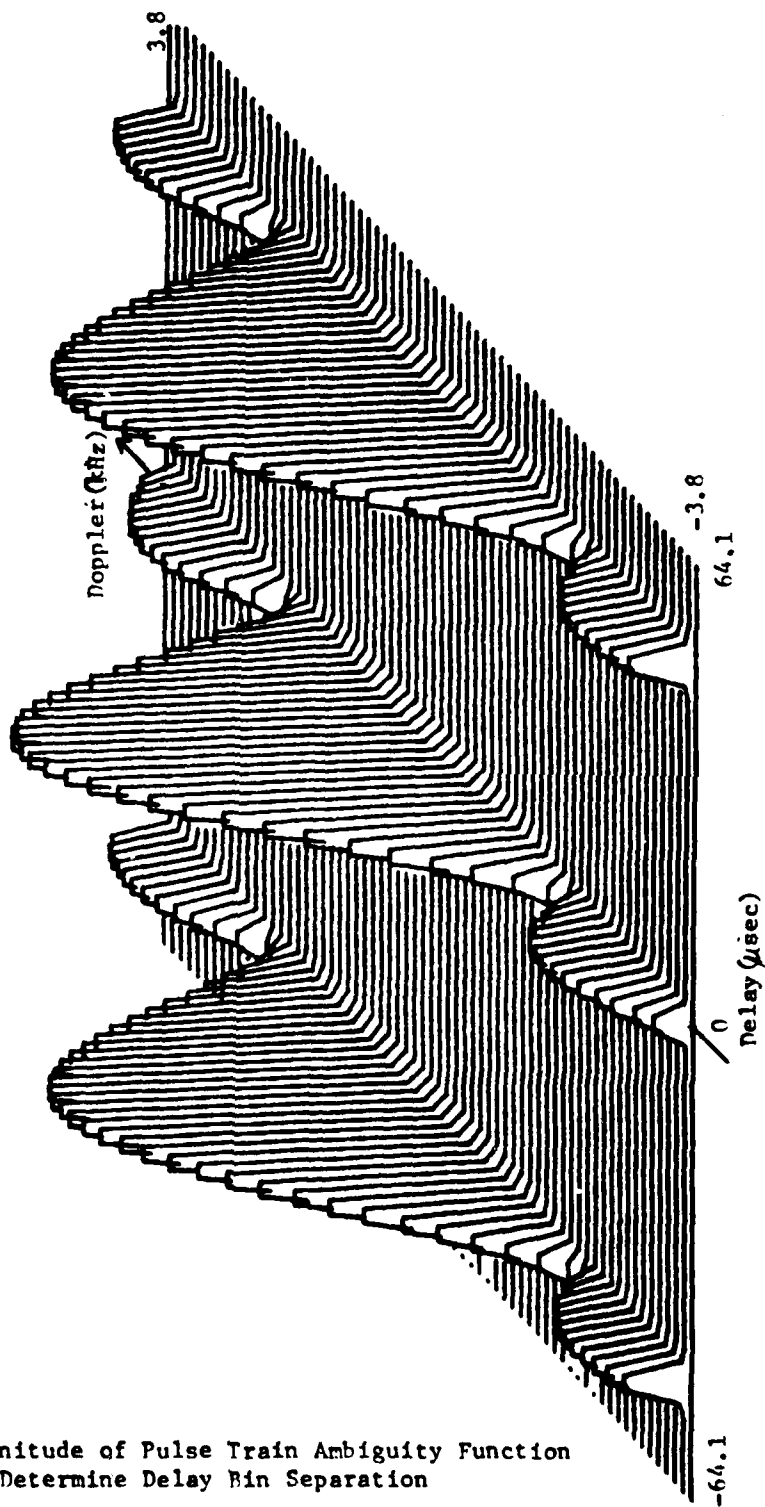


Fig 5. Magnitude of Pulse Train Ambiguity Function  
to Determine Delay Bin Separation

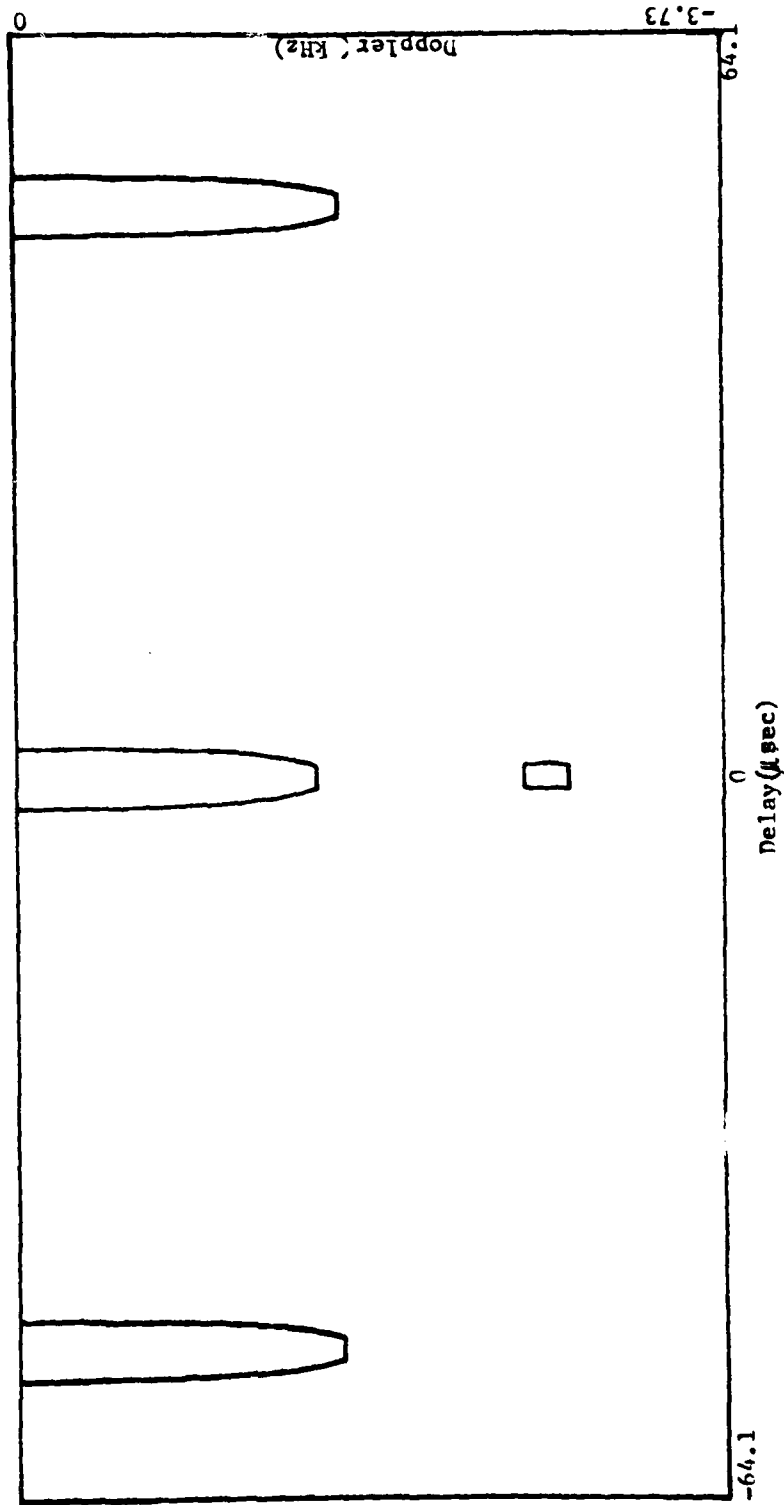


Fig 6.  $-3\text{dB}$  Contour of Magnitude Squared of the Pulse Train Ambiguity Function to Determine Strength of Subsidiary Peaks

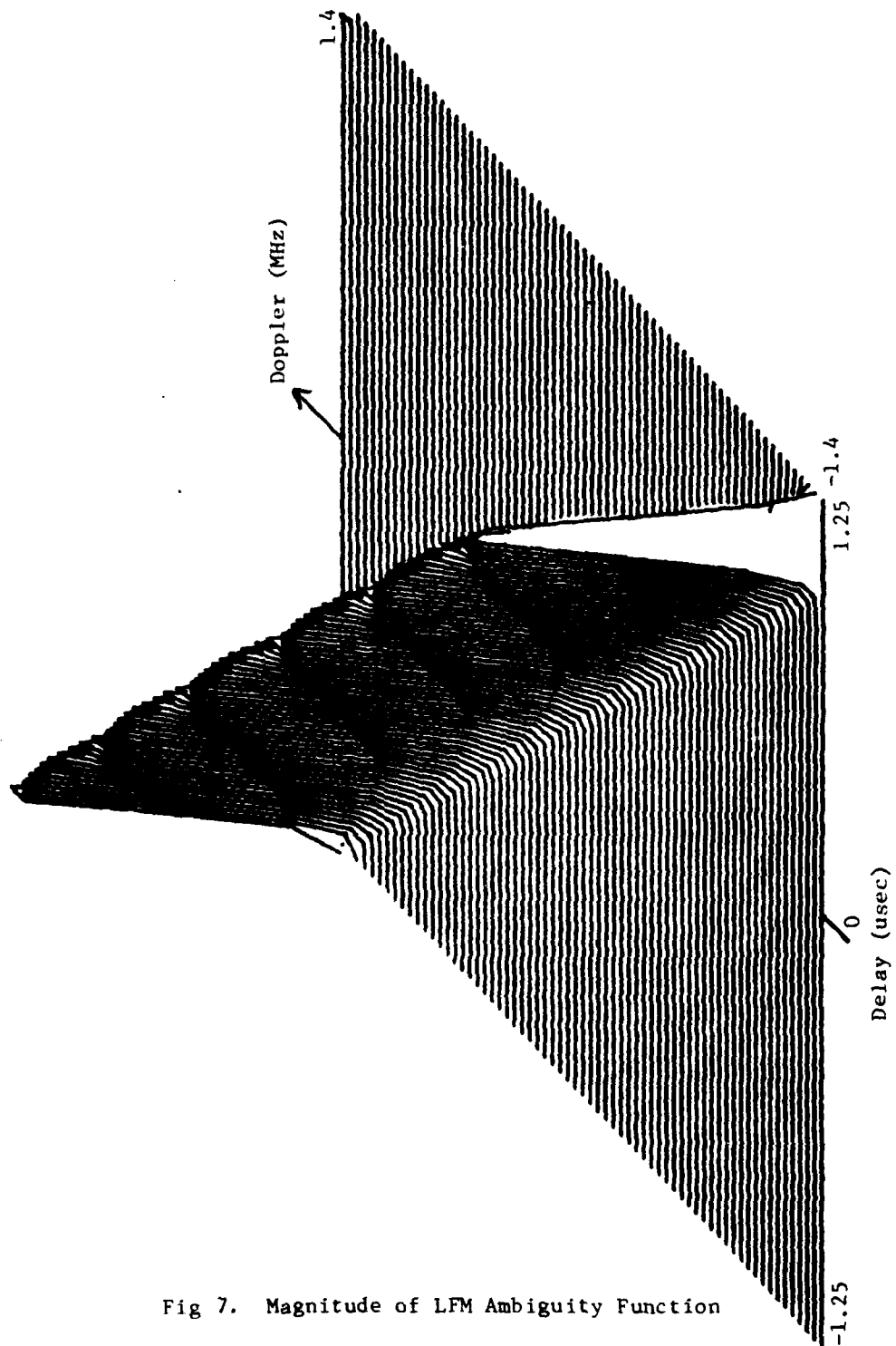


Fig 7. Magnitude of LFM Ambiguity Function

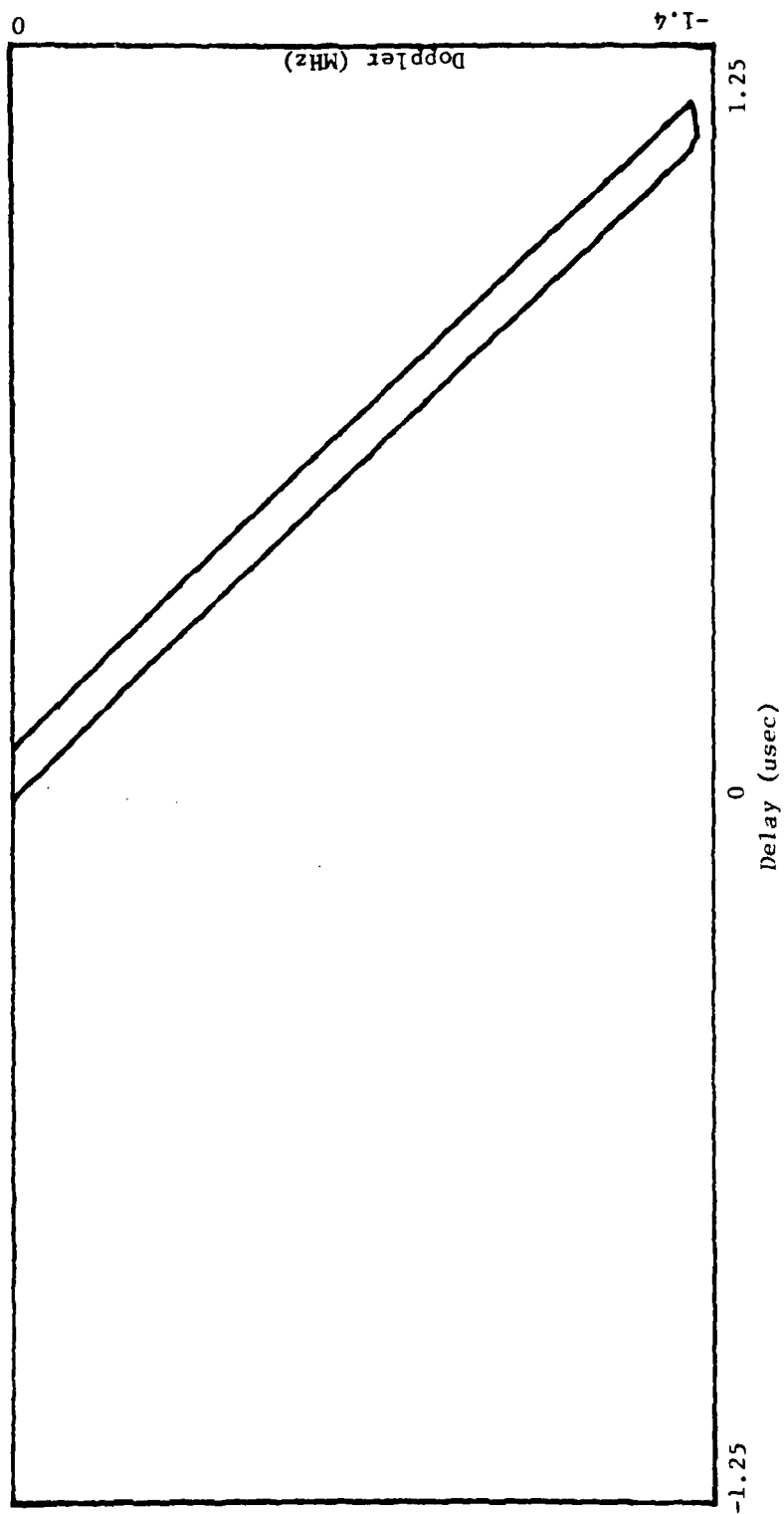


Fig 8. -3dB Contour of Magnitude Squared of the LFM Ambiguity Function



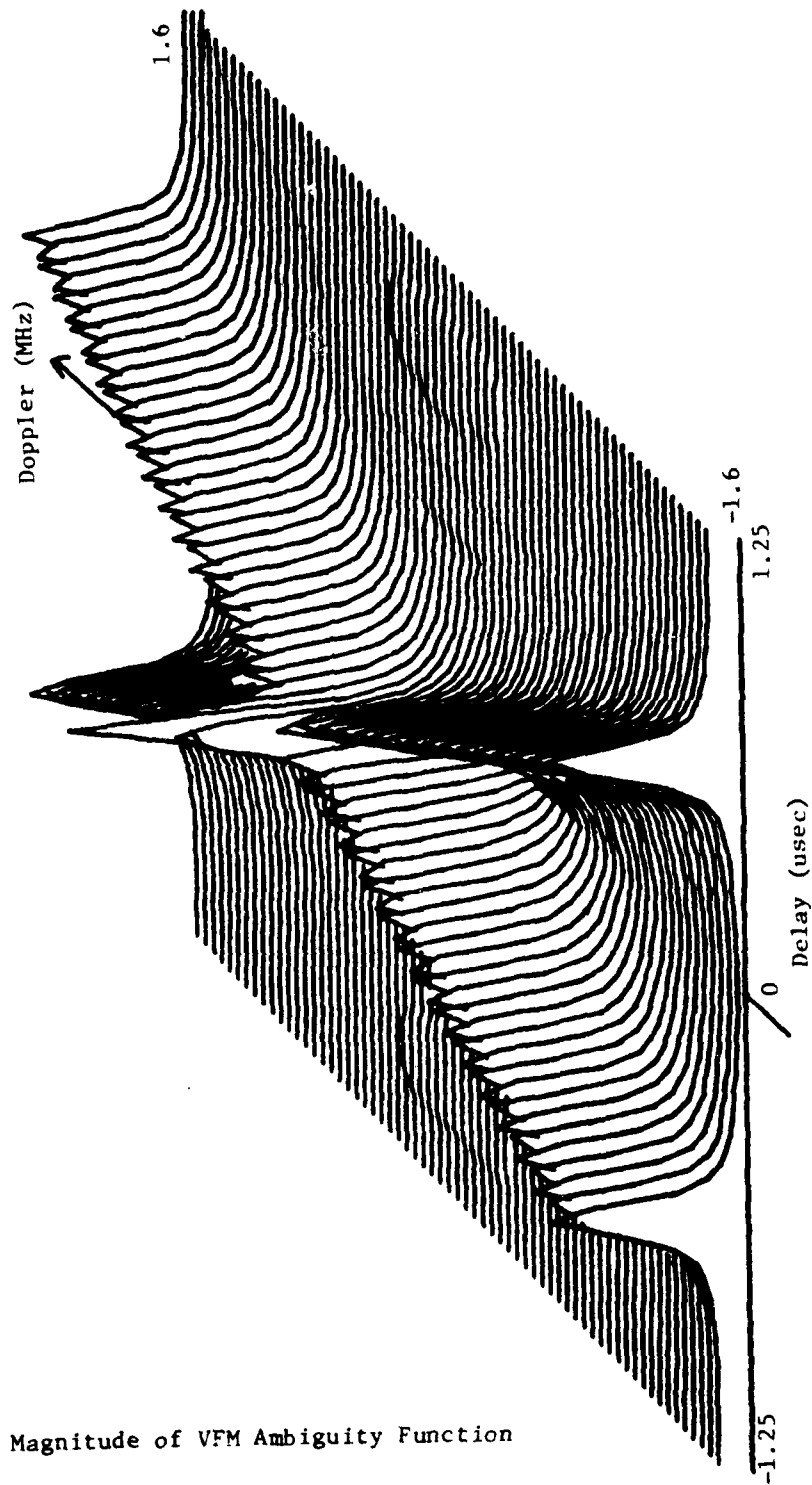


Fig 9. Magnitude of VFM Ambiguity Function

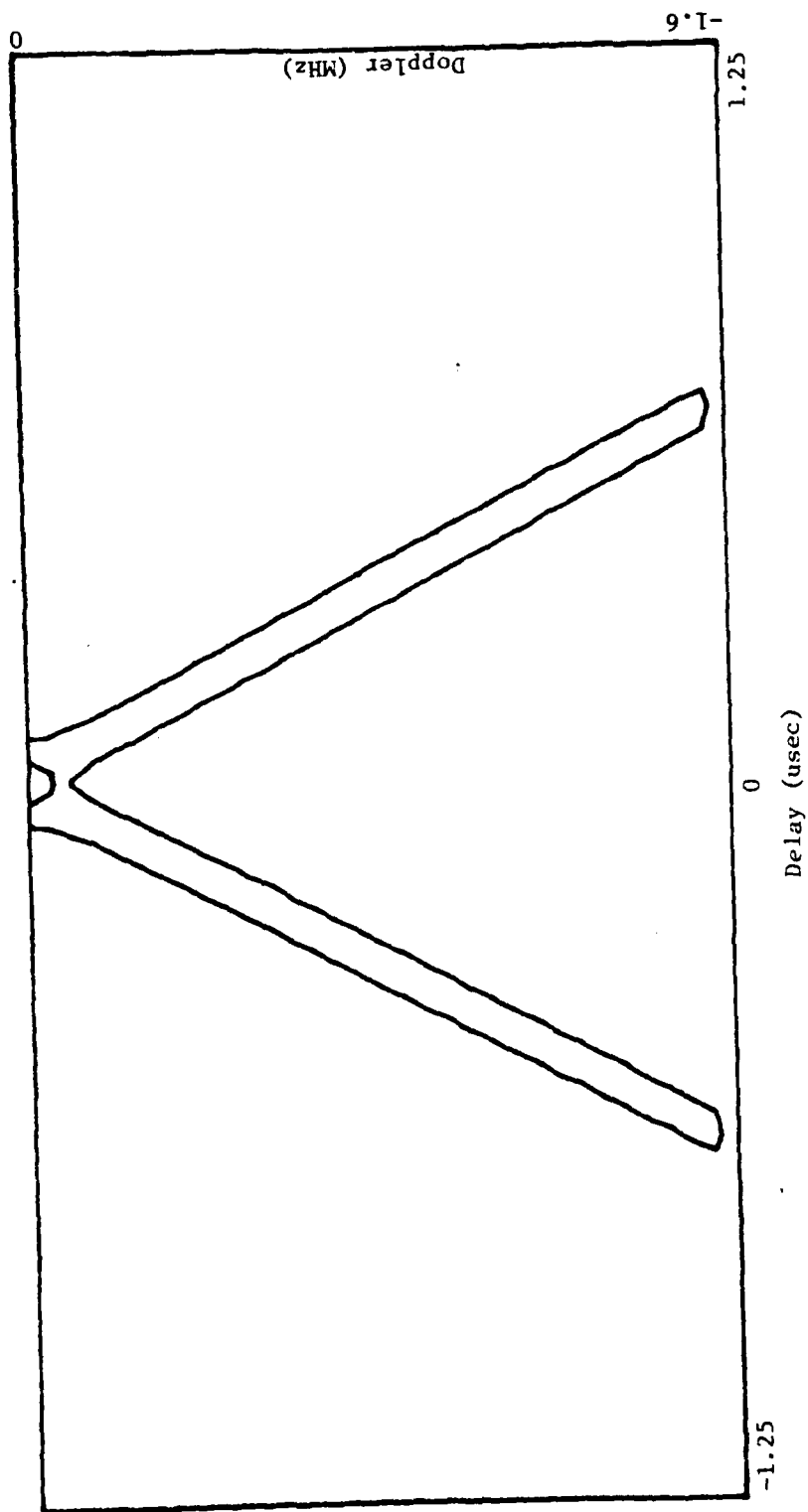


Fig 10. -3dB and -6dB Contours of Magnitude Squared of the VFM Ambiguity Function

Since we assume a coastal environment, the radar will receive sea clutter, ground clutter, and occasionally weather clutter. We will want to have a reasonably accurate model for all three types of clutter.

The first clutter model we will develop will be the one for the sea clutter. From the discussion of clutter previously provided in this study, we know that the spectrum of sea clutter in velocity is approximately a linear function of wind speed. The user must determine the average wind speed for the general location of the radar. For our purposes, we will assume an average wind speed of 4 m/sec (corresponding to 80 Hz). From Figure 1 we find the width of the Doppler spectrum to be:

$$\frac{2(2.3\text{m/sec})(3 \times 10^9 \text{sec}^{-1})}{(3 \times 10^8 \text{m/sec})} = 46 \text{ Hz}$$

We also assume the sea clutter to be distributed as Gaussian with zero mean and standard deviation of 15 Hz. We also assume the sea clutter to be distributed uniformly in range from the radar origin out to the radar horizon (assume it to be 6 km). On the delay axis, this would correspond to clutter from 0 to 10  $\mu\text{sec}$ . Hence the joint probability density function would be

$$p(\tau, V) = \frac{1}{20\mu\text{sec}} \left\{ \frac{1}{\sqrt{2\pi}(15)^2} \exp - \left( \frac{V}{2(15)^2} \right)^2 \right\}$$

We now want to determine the average cross section of the sea clutter and estimate the number of scatterers from sea clutter in the radar beam at any time. Since the waveforms under consideration have pulse widths of at least 5  $\mu\text{sec}$ , the sea clutter amplitude can be modeled by a Gaussian or Rayleigh envelope model. The mean cross-sectional area can then be determined by using Eq 72. From Figure 11 and assuming a grazing angle of  $10^\circ$ ,  $\sigma_0$  can be approximated to -42dB for horizontal polarization and -35dB for vertical polarization. Since we will use horizontal polarization, we assume  $\sigma_0 = -42\text{dB}$ . We can approximate  $A_C$  by the equation

$$A_C = R\theta_B(C\tau/2)\sec \phi \quad (84)$$

where

$R$  = range of clutter

$C$  = velocity of propagation

$\theta_B$  = azimuth beamwidth

$\phi$  = grazing angle

$\tau$  = pulse width (Ref 11:472).

Assuming an azimuth beamwidth of  $3^\circ$  and an average range of 3 km, we approximate  $A_C$  as  $1.2 \times 10^5 \text{m}^2$  for the 5  $\mu\text{sec}$  pulse and  $4.8 \times 10^5 \text{m}^2$  for the 20  $\mu\text{sec}$  pulse. Hence the average cross section of the sea clutter will be approximately  $7.6 \text{m}^2$  for the 5  $\mu\text{sec}$  pulse and  $30.3 \text{m}^2$  for the 20  $\mu\text{sec}$  pulse. We estimate that we would receive 10 returns from sea clutter

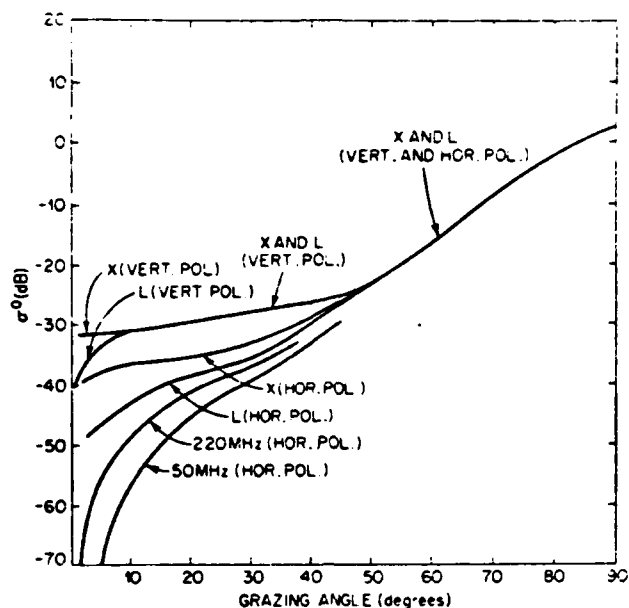


Fig 11. Composite of  $\sigma^0$  data for average conditions with wind speeds ranging from 10 to 20 knots

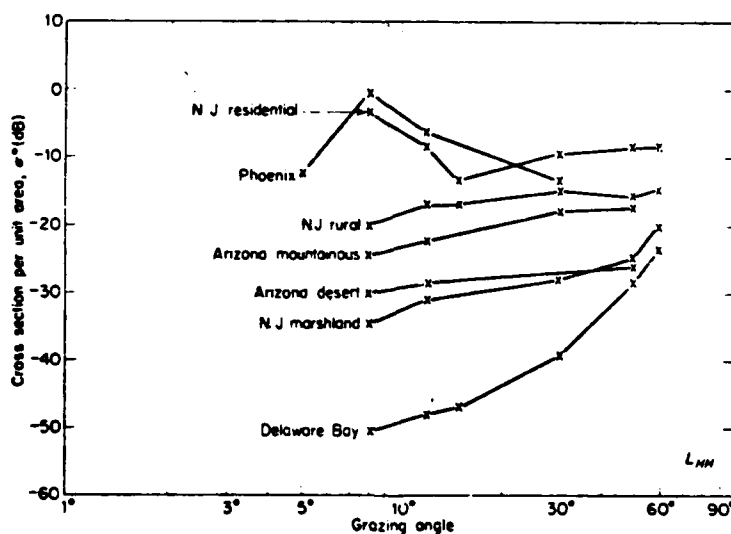


Fig 12. L Band, Horizontal Polarization Values of  $\sigma^0$  for Various Environments (Ref 11:475,493)

at the same time we would want to detect the target.

We next want to consider the ground clutter. Assuming the area will be flat and uncultivated, we can use the graphs Figure 12 to estimate  $\sigma^0 = -42\text{dB}$  (Ref 11:495). Hence our average clutter returns over land will be similar to those for sea clutter.

To determine the distribution of ground clutter, we recall that the Doppler spectrum will be centered at zero. For a wind speed of 10 knots, we estimate the standard deviation of the Doppler to be 0.125 m/sec (which corresponds to 2.5 Hz). Again we assume the clutter to be uniformly distributed in range from the radar origin out to the radar horizon. Hence the probability density function for the ground clutter in terms of delay and Doppler is:

$$p(\tau, V) = \frac{1}{20\mu\text{sec}} \left\{ \frac{1}{\sqrt{2\pi}(2.5)^2} \exp \left( -\frac{V^2}{2(2.5)^2} \right) \right\}$$

Finally, we want to consider the effect of weather clutter on the detection performance. To model the weather clutter, the user will need weather statistics from the general operational area. If we assume rain showers with a rainfall rate of 4 mm/hr occur 5 percent of the time, the clutter amplitude returns can be modeled by Gaussian (or Rayleigh envelope) probability density function. The average cross sectional area can be estimated at approximately  $10^{-3} \text{ cm}^2/\text{m}^3$  (cross section per unit volume) (Ref 11:501).

Since the range of weather clutter can be from the radar origin out to the maximum range of 50 km, very large cumulative clutter returns can be expected. We assume an average return of  $0.04\text{m}^2$  with 1000 returns in a beamwidth.

Assuming a mean wind speed of 6 m/sec (approximately 18 mph), the mean of the weather clutter velocity will be similar. From our previous discussion of weather clutter, we know  $\sigma_{\text{turb}} \approx 1.0$  m/sec and, assuming an average range of 20 km and a velocity gradient component of 4.0 m/sec/km,  $\sigma_{\text{shear}} \approx 1.0$  m/sec. Therefore, the total spectrum width,  $\sigma_v$ , will be about 1.0 m/sec. Again we assume a uniform distribution in range from the radar origin out to 40 km (corresponding to 67  $\mu\text{sec}$ ). Hence the clutter probability density function can be modeled as:

$$p(\tau, V) = \frac{1}{134\mu\text{sec}} \left\{ \frac{1}{\sqrt{2\pi}(5)^2} \exp \left( -(V-30)^2 / (5)^2 \right) \right\}$$

By using the clutter plot program, we now can plot the clutter distributions as functions of delay and Doppler. A plot of the sea clutter distribution is shown in Figure 13. Plots of the ground clutter and weather clutter are shown in Figures 14 and 15, respectively. Also included are plots (near the clutter regions) of the magnitude of the ambiguity functions the repeated pulse, LFM, and VFM waveforms in Figures 16, 17, and 18, respectively. A cursory examination of the ambiguity function magnitudes indicates that the LFM waveform

should have the best signal-to-clutter ratio since its delay width is much narrower than the repetitive pulse, and since the VFM waveform has the pedestal-like magnitude away from the ridge.

Now we use the signal-to-clutter program to determine quantitatively how each waveform will perform in the environment. We assume the smallest desired target has a radar cross section of  $4\text{m}^2$ . When sea clutter was considered, the signal-to-clutter ratio was calculated to be 0.5 for the repetitive pulse waveform, 2.7 for the LFM waveform, and 1.9 for the VFM waveform. The signal-to-clutter ratio over the ground clutter was calculated to be 0.5 for the repeated pulse waveform, 2.7 for the LFM waveform, and 1.9 for the VFM waveform. Finally, the signal-to-clutter ratio over the weather clutter was calculated to be 2.4 for the repeated pulse waveform, 139 for the LFM waveform, and 97.9 for the VFM waveform.

Thus, we find that none of the candidate waveforms provide the necessary signal-to-clutter ratio to detect a  $4\text{m}^2$  target over the sea and ground clutter areas. The LFM and VFM waveforms would allow detection of a  $4\text{m}^2$  target in the weather clutter, while the repeated pulse waveform would not. The user must now consider making tradeoffs between the waveforms and requirements.

If the user is willing to sacrifice coverage over the clutter areas, then the repeated pulse waveform should be



chosen since it exceeds both the delay and Doppler resolution requirements. If coverage over the clutter areas is more critical than resolution requirements, then the user will choose the LFM waveform. A much better signal-to-clutter ratio can be obtained by using an LFM waveform with a 5  $\mu$ sec pulse width and a frequency modulation band of 100 MHz (see Figure 19). With this waveform, the signal-to-clutter ratio would increase to 43.8, 43.8, and 55.7 over the sea, ground, and weather clutter, respectively. The delay-Doppler ridge would be narrowed in delay, but would be extended in Doppler. Thus, the user must determine what requirements are the most important and then make tradeoffs in the requirements to determine a waveform for the radar system.

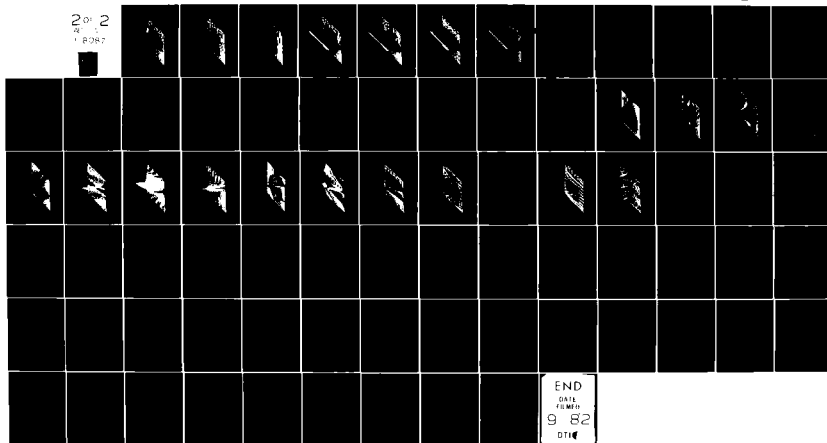
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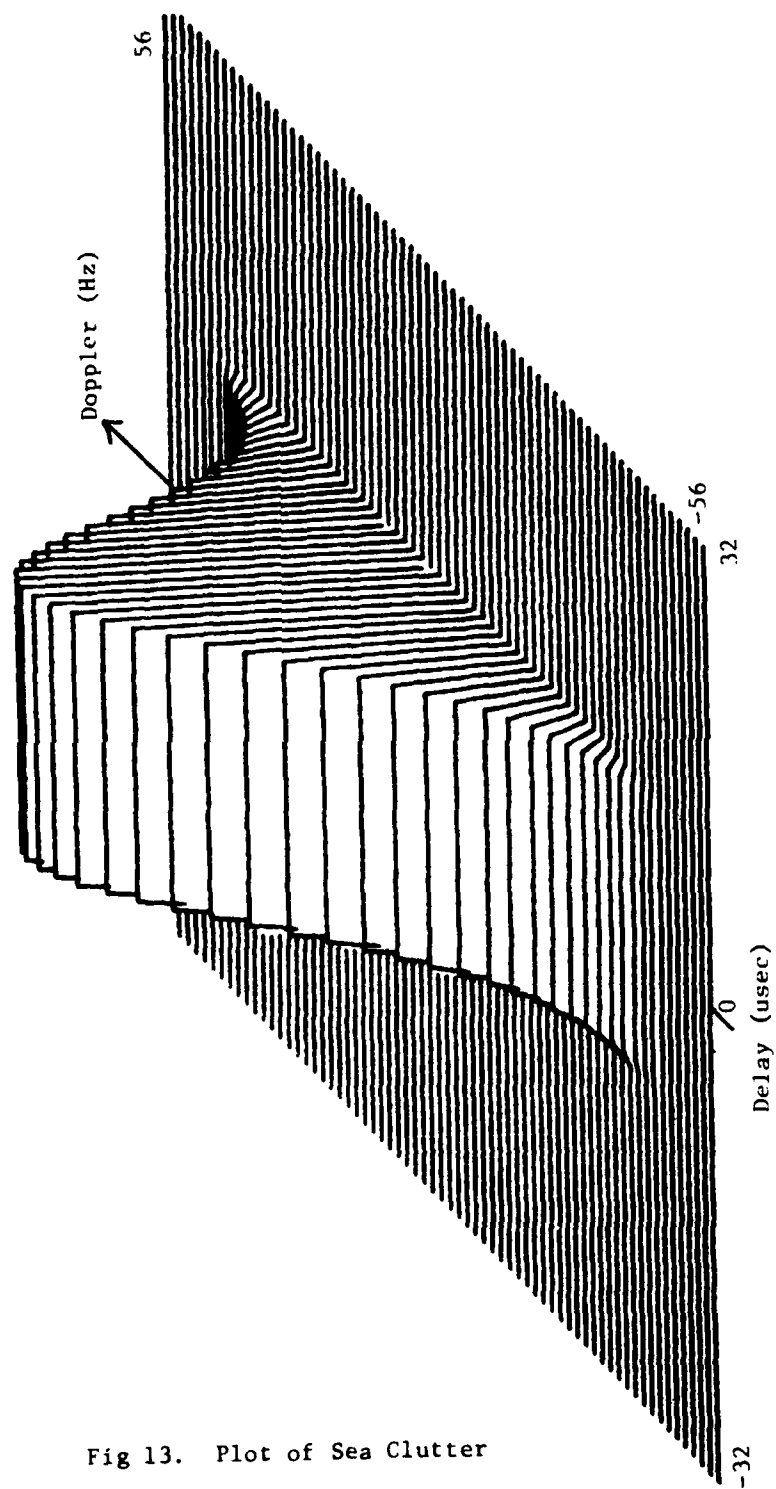


Fig 13. Plot of Sea Clutter

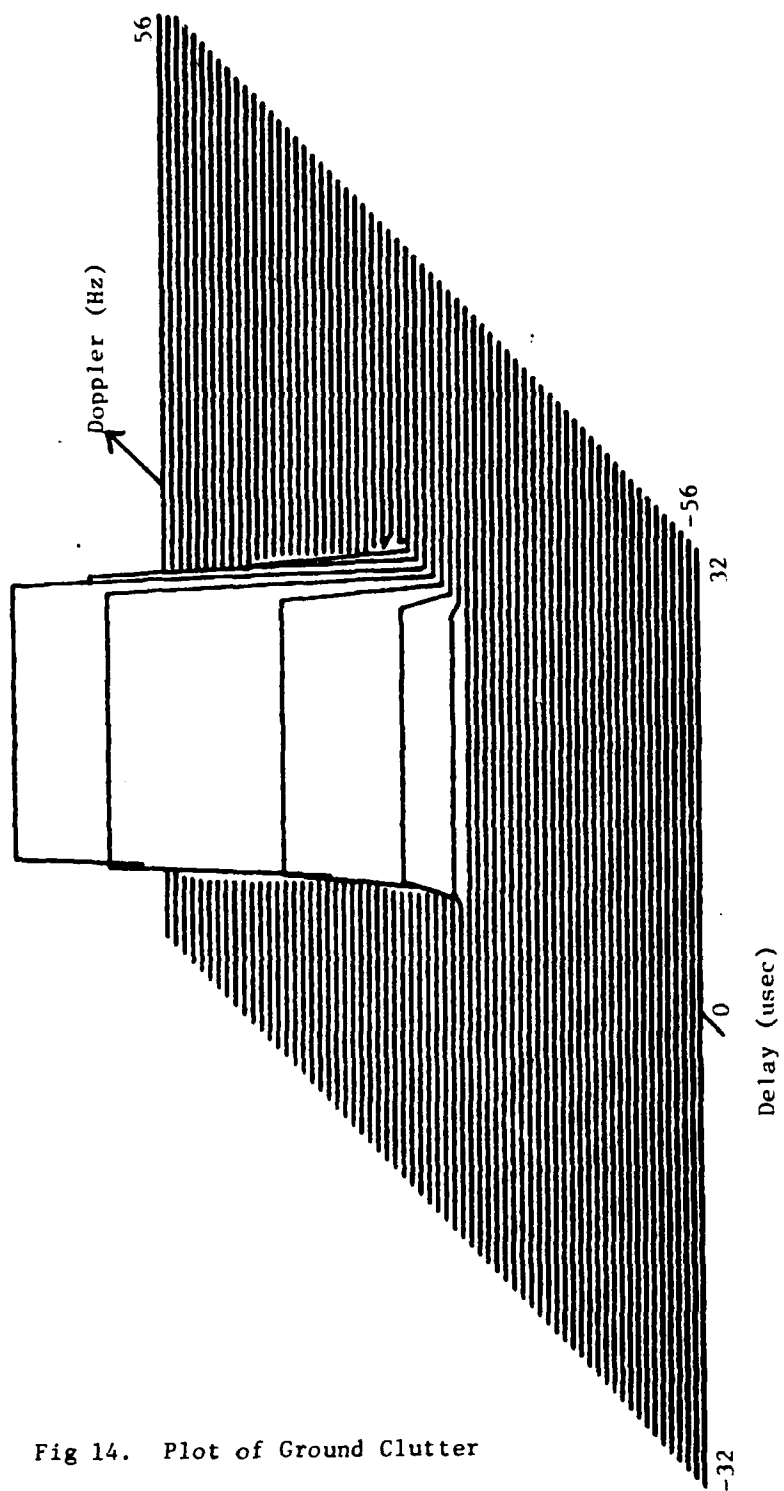


Fig 14. Plot of Ground Clutter

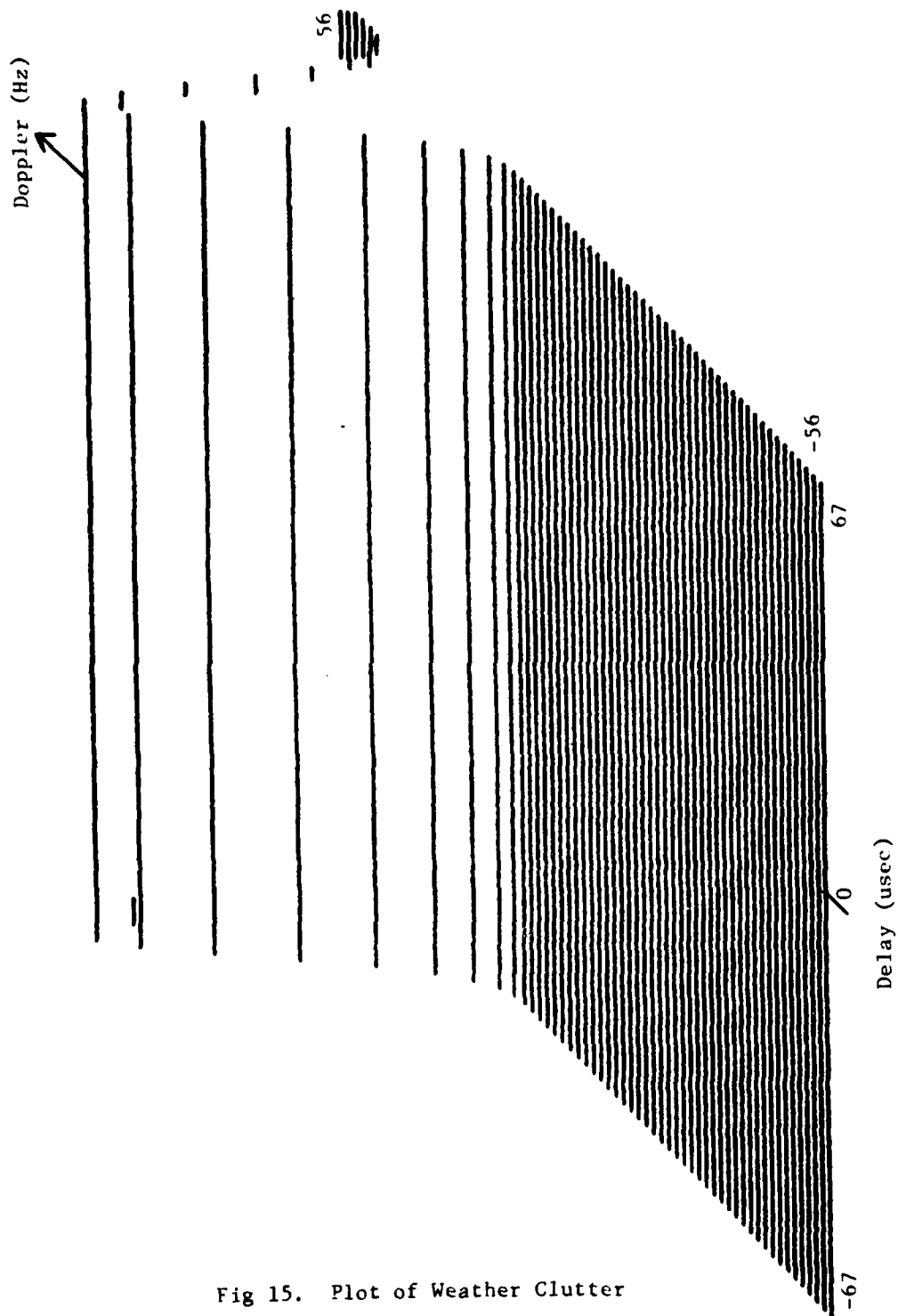


Fig 15. Plot of Weather Clutter

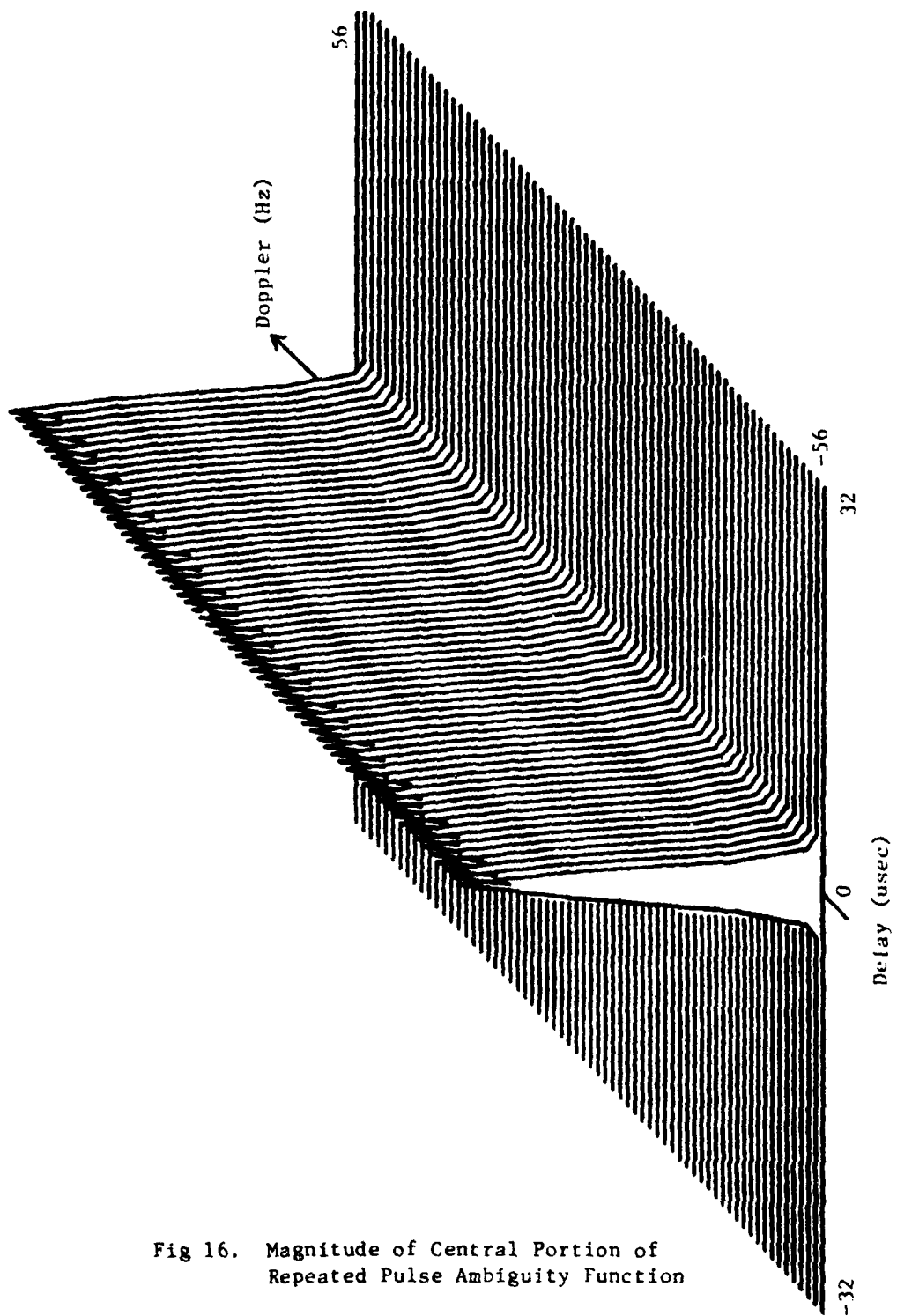


Fig 16. Magnitude of Central Portion of Repeated Pulse Ambiguity Function

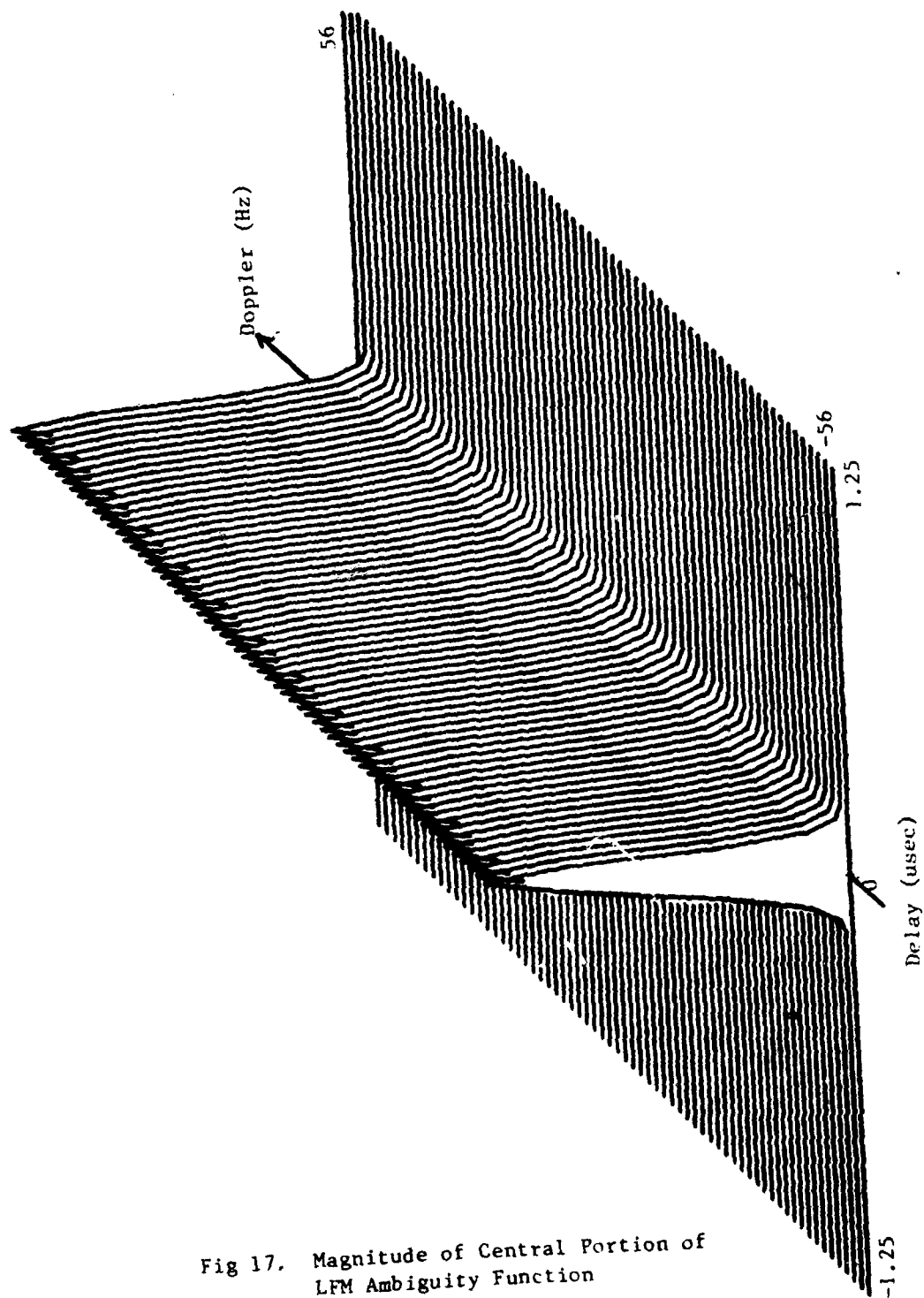


Fig 17. Magnitude of Central Portion of LFM Ambiguity Function

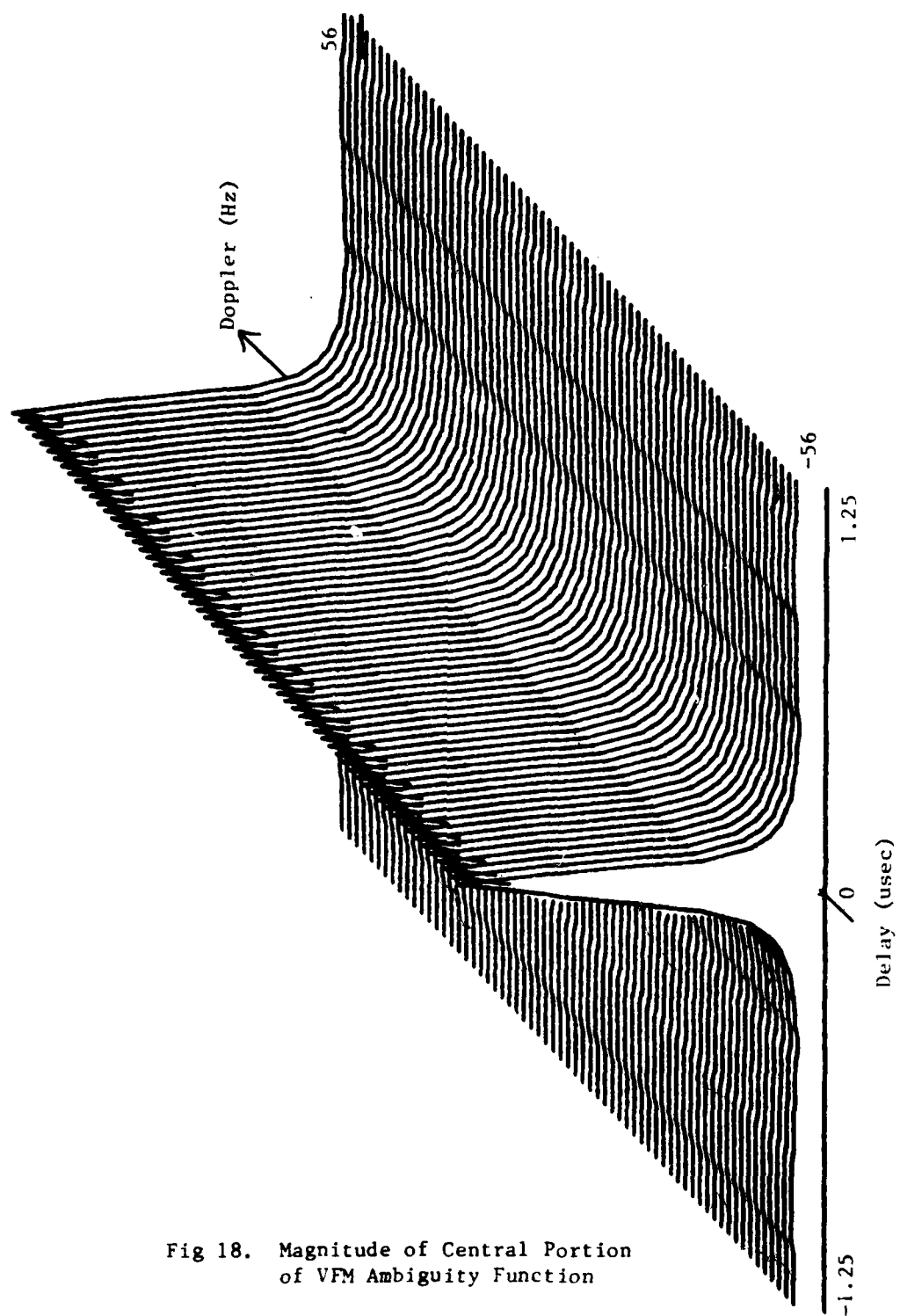


Fig 18. Magnitude of Central Portion  
of VFM Ambiguity Function



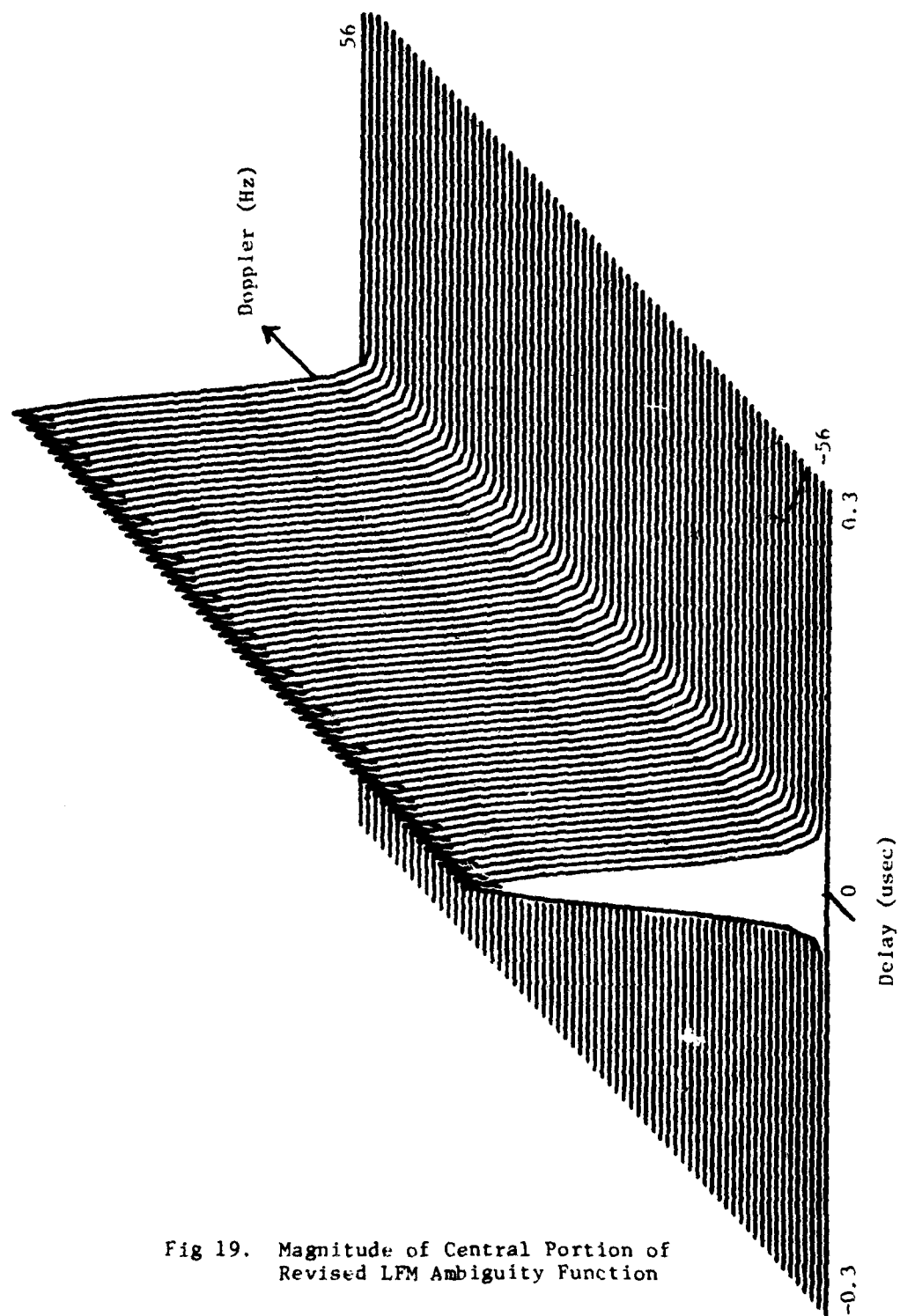


Fig 19. Magnitude of Central Portion of Revised LFM Ambiguity Function

## V. Conclusions and Recommendations

### Discussion

As stated at the beginning of this paper, this study was to develop a fast, simple, and accurate method of selecting a radar waveform when provided the target detection requirements and the expected operating environment. The majority of work for this study was involved in developing software to calculate the magnitude of radar ambiguity functions, to depict these magnitudes in three-dimensional diagrams, and to calculate the signal-to-clutter ratio of a desired target in the modeled environment. This work was limited to considering only the most common waveforms used in radar systems; however, these waveforms provided the basis for more complex waveforms.

### Conclusions

From the work done and presented in this study, the method proposed for waveform selection provides the fast, simple, and accurate technique desired. By calculating and analyzing the magnitude of ambiguity functions and the signal-to-clutter ratio, the waveform selection process becomes simpler. The discussion of modeling the clutter environment describes how to reasonably model the projected operating environment for the waveform selection process. Also, the discussion of basic resolution properties of waveforms allows

the user to select only those waveforms with a reasonable probability of success for detailed analysis.

Once a list of candidate waveforms is selected, the three-dimensional diagram of the ambiguity function magnitude (as a function of delay and Doppler) for each waveform allows the user to insure the resolution and ambiguity requirements are fulfilled. With proper sampling, these magnitudes are within one percent of the actual results. For those waveforms fulfilling these requirements, the signal-to-clutter ratio for desired targets is calculated for each waveform. The discussion on tradeoffs in this paper describes how the user should continue the selection process when several waveforms fulfill the requirements or when none of the waveforms fulfill the requirements.

To use this method of waveform selection, the user must have access to a computing system with a high-level language capability, at least 32K bytes of memory, a fast Fourier transform program, and a three-dimensional graphics program. All software developed in this study will provide accurate results as long as the proper parameters are input to the program. The user must have a basic understanding of radar theory and digital signal processing techniques to successfully use the software. A brief discussion of both topics is included in this paper.

### Recommendations

As stated previously, this study was limited to waveforms generally used in contemporary radars. Ambiguity functions using azimuth angle and elevation angle were also not considered. The study also only considered the case where there was negligible distortions of the complex envelope of the waveform by target motion, target acceleration, and higher order derivatives.

Many other radar waveforms exist other than those considered in this study. As technology improves, the cost to implement these waveforms in radar systems will decrease and make their use more appealing. Thus, further study of additional waveforms is suggested.

The ambiguity functions calculated using azimuth angle and elevation angle can significantly contribute to the design and selection of antennas for radar systems. All required theory to calculate these ambiguity functions exists, and therefore, a method for selecting an antenna for the radar can be developed similar to the waveform selection method of this study.

Although there is a wide range of practical applications in which target motion does not distort the waveform, situations exist in which a radar must detect high-speed targets. Extension of the theory for such situations exist, but the theory becomes much more complex. Consideration should be

given to developing a method of waveform selection when high-speed targets must be detected.

An improved three-dimensional graphics program would improve the waveform selection process. Because the graphics program used in this study was very limited in the number of samples it could display, the resolution and range of the three-dimensional diagrams was only adequate. With a program which could plot  $256 \times 256$  arrays, the use of these diagrams would increase and fewer diagrams would be required in analysis.

Finally, the performance of receivers which do not use a matched filter can still be evaluated. The cross-ambiguity function can be determined and used in the waveform selection process. This study made no provision to calculate the cross-ambiguity function; however, the development of software to consider cross-ambiguity functions could follow directly from that for the ambiguity function. Hence, the cross-ambiguity function software should be developed.

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## APPENDIX A

### Operation of Software on the Eclipse Computer

The basic principles of operating the Eclipse computer are explained so that the following items can be produced: the magnitude of cuts of ambiguity functions, three-dimensional diagrams of the magnitude of the ambiguity function, and the signal-to-clutter ratio. The Eclipse uses the Command Line Interpreter (CLI) as the basic operating command set. When the Eclipse can accept a command, an 'R' prompt appears on the terminal. The user should enter the command, DIR DP4: REED <CR>, which puts the user in the directory containing the software programs.

The user should enter the command, MAIN <CR>, to run the program which simply provides the magnitudes of the ambiguity function for a particular Doppler cut. To calculate the signal-to-clutter ratio, the user should enter the command, IMAIN <CR>. When a three-dimensional plot of the magnitude of the ambiguity function is desired, the user should enter the command, GMAIN <CR>. For a three-dimensional plot of the clutter probability density function, the user should enter the command, CLUTR <CR>. After GMAIN or CLUTR runs, the user will then have to use the Textronix storage scope to observe the three-dimensional plot. Once the control box for the foreground is set to the Textronix scope, the user should enter the command, PLTTRNS DATA/I Num/N <CR> for the



ambiguity function magnitude plot or PLTTRNS CLUTTER/I 64/N for the clutter plot. The value of Num will be specified at the end of the GMAIN program, and the user must note it to enter when PLTTRNS is run.

When MAIN, GMAIN, IMAIN, or CLUTR is run, the user will be required to make entries during the program. These entries will specify the data to enter, and all data should be entered in decimal notation. A carriage return, <CR>, is required after all entries. If the user makes a mistake in the entry while using the Background terminal, the DEL key may be used to erase characters. The '/' key may be used to delete the entire entry on both the Foreground and Background terminals. The user may enter '^A' at any time to terminate the program being run. When a program has ended, the user will see an 'R' prompt on the terminal.

When PLTTRNS is run, the user must use the Textronix scope. This program will also require the user to make entries. If the entry is an alphabetic character, no <CR> is required. After any numerical entry, the user must use a <CR> to indicate the end of the number. If a copy of the plots on the scope is desired, the user simply depresses the COPY key on the Textronix keyboard. The user should insure the copier has been on for at least ten minutes before copies are made. The diagrams produced by PLTTRNS do not have the axes identified or labeled. The user can obtain the delay and Doppler limiting values from the GMAIN or CLUTR program.

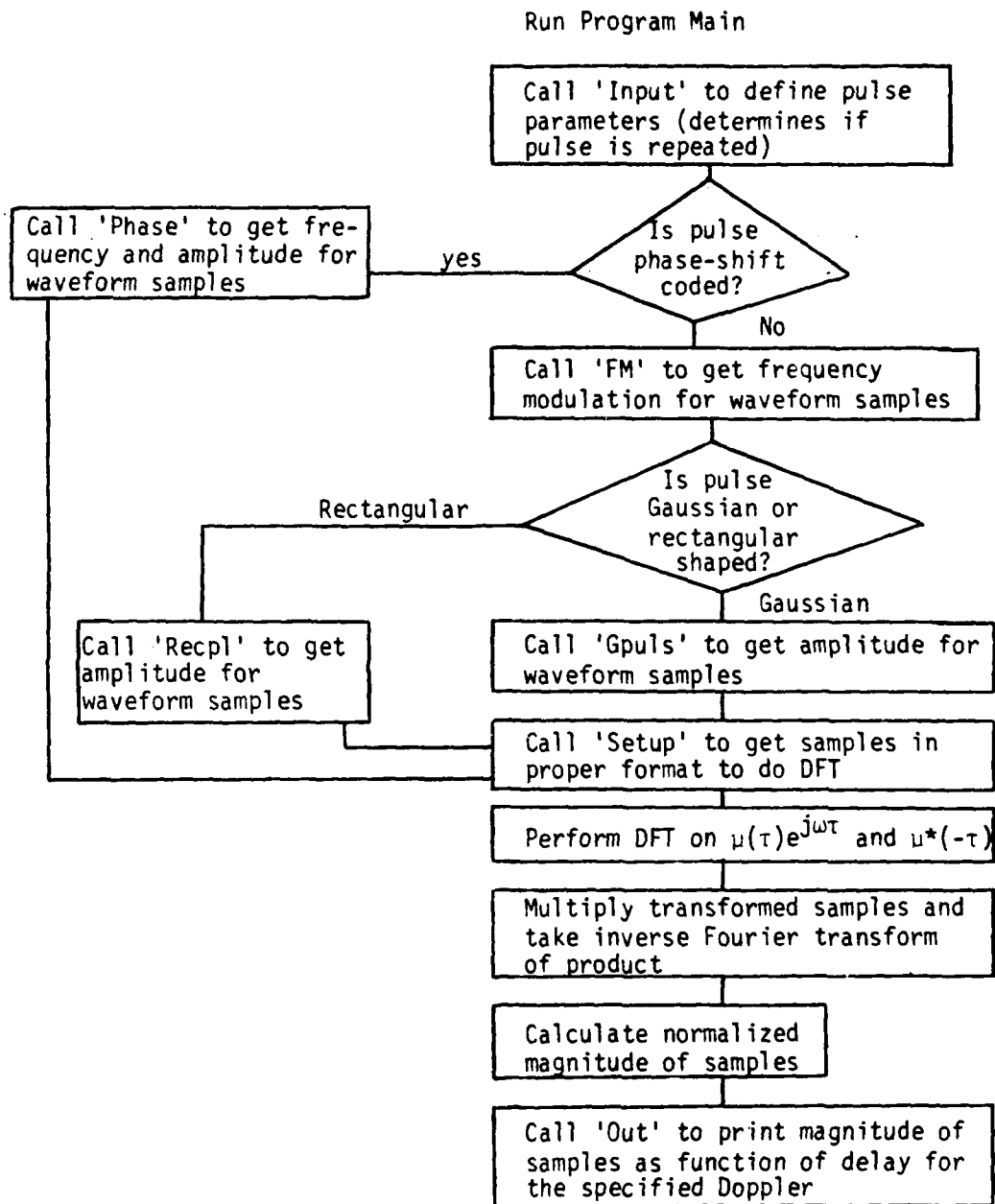
If copies of the program listings are desired, the user should enter the command, 'PRINT filename.FR, filename.FR,... filename.FR'. The program listings indicate all the subroutines called, and these are the filenames to enter along with the program name.

After the user has completed the use of all the programs, the computer should be returned to the DPØ directory. This is done by entering the command, DIR DPØ <CR>.

APPENDIX B

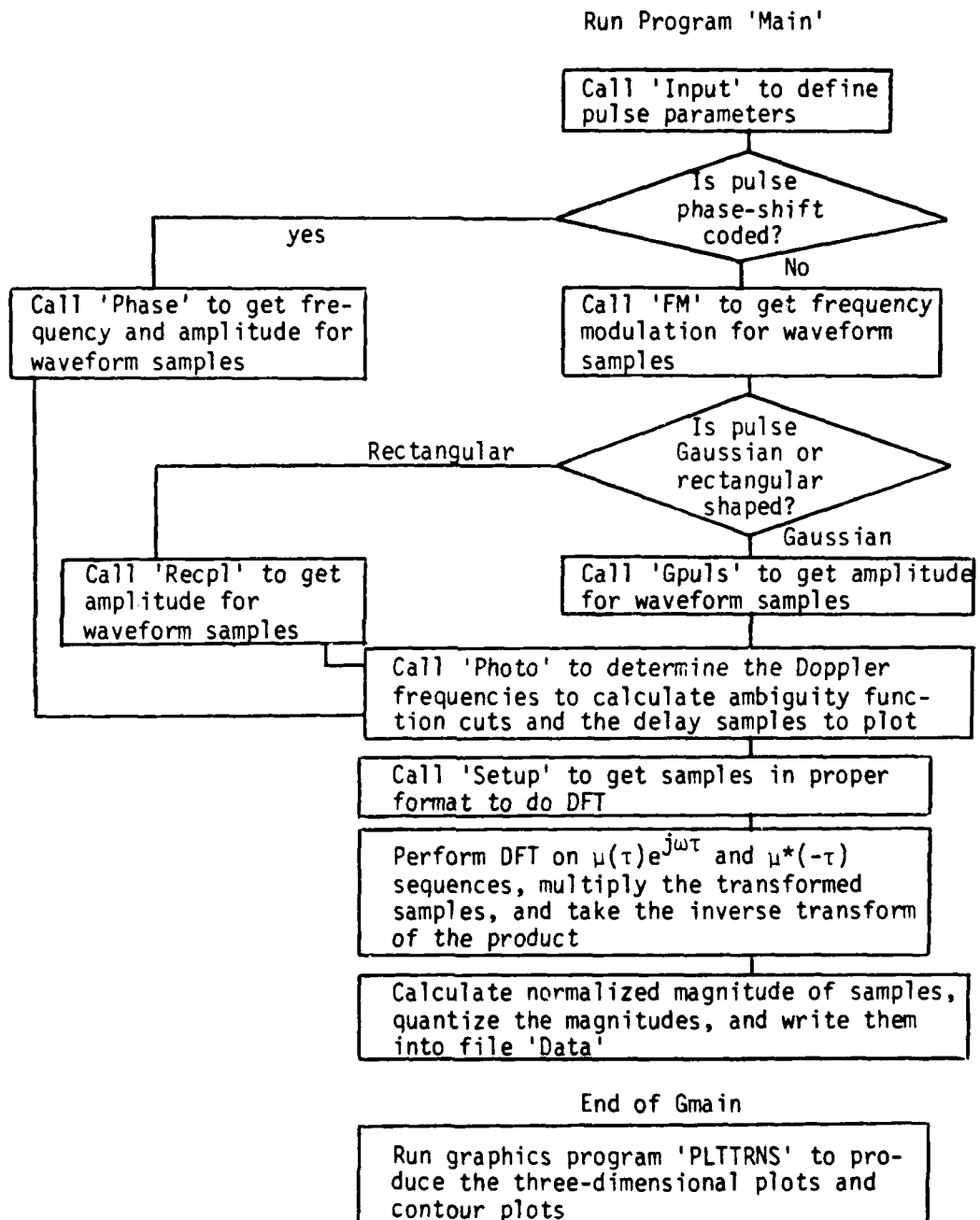
System Flowchart

# Flowchart for Calculating Magnitude of Ambiguity Function

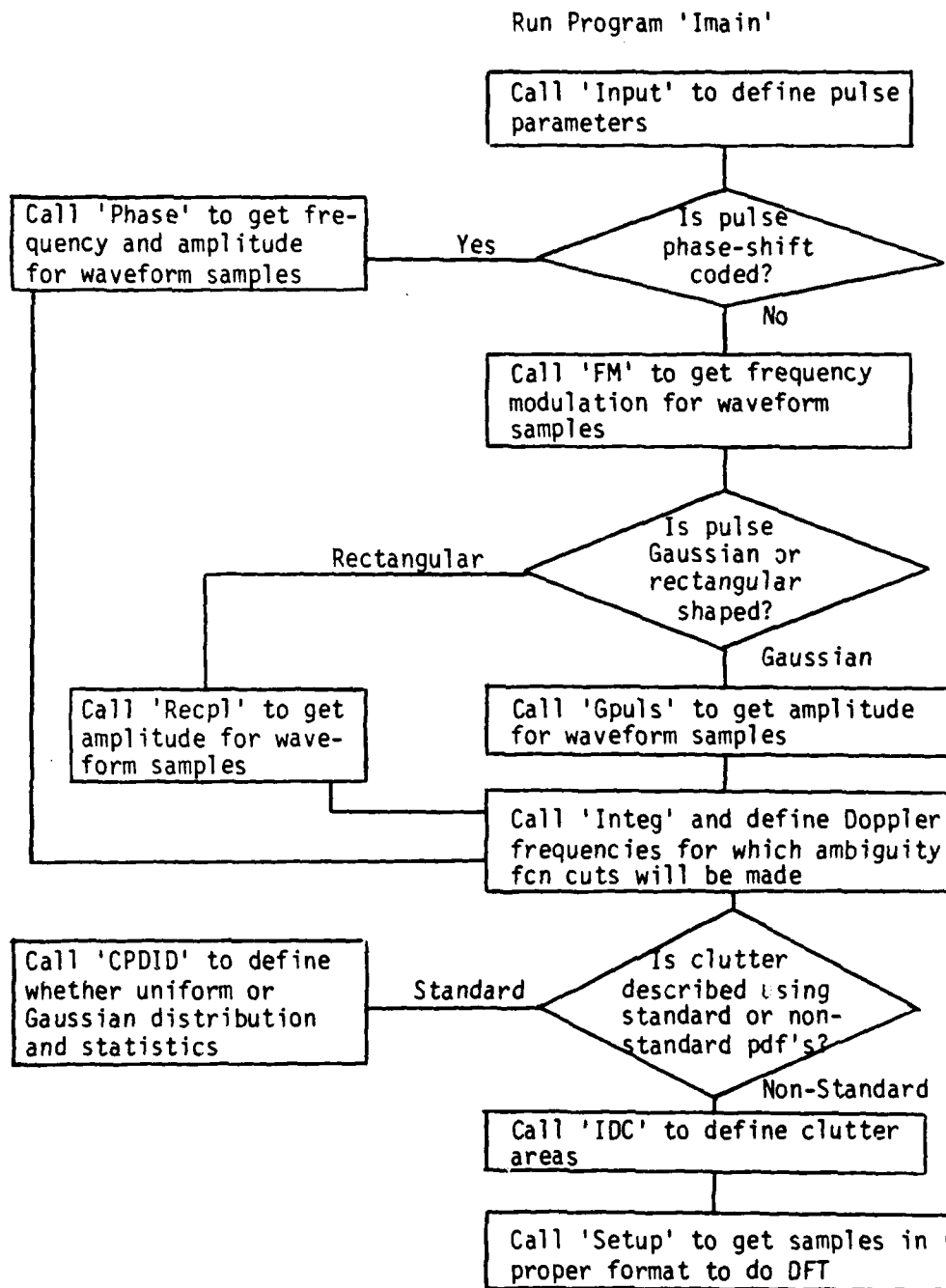


END

# Flowchart for Three-Dimensional Diagram of Ambiguity Function Magnitude

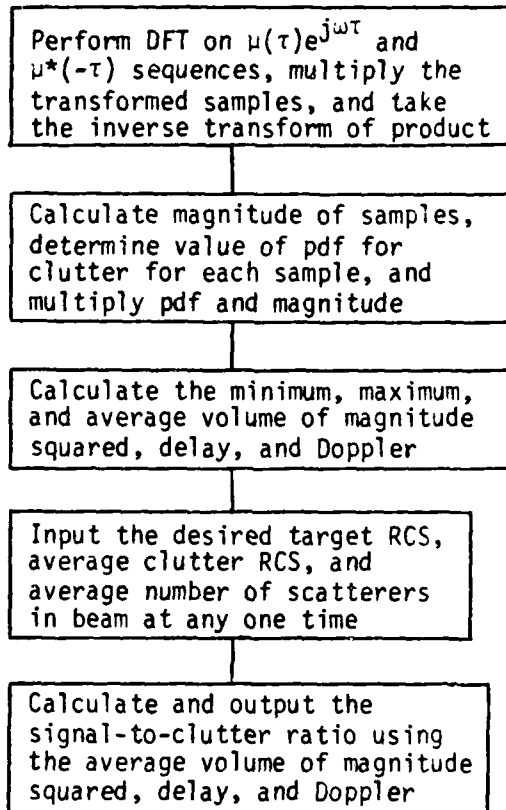


# Flowchart for Calculating Signal-to-Clutter Ratio



Continued

Continuation



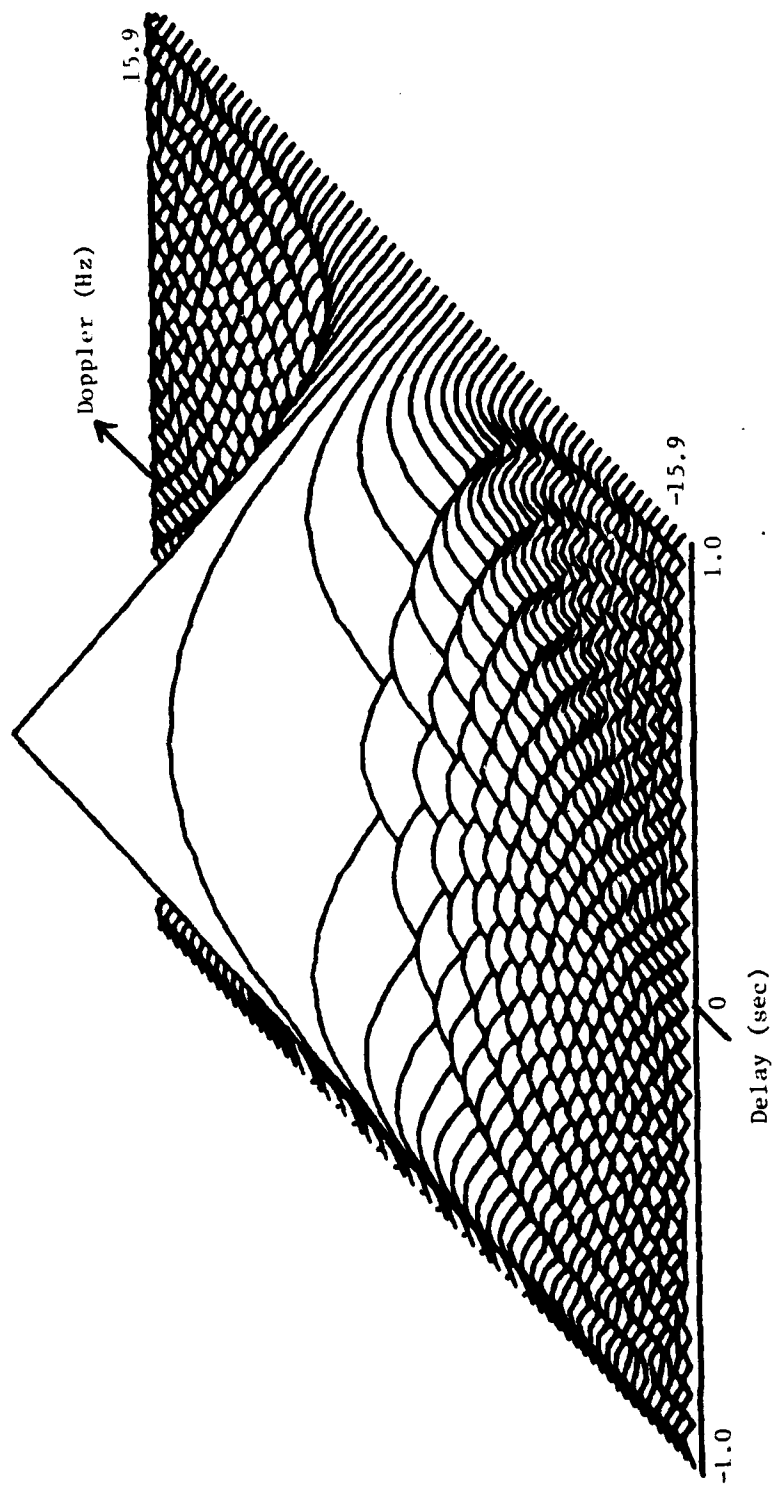
END

## APPENDIX C

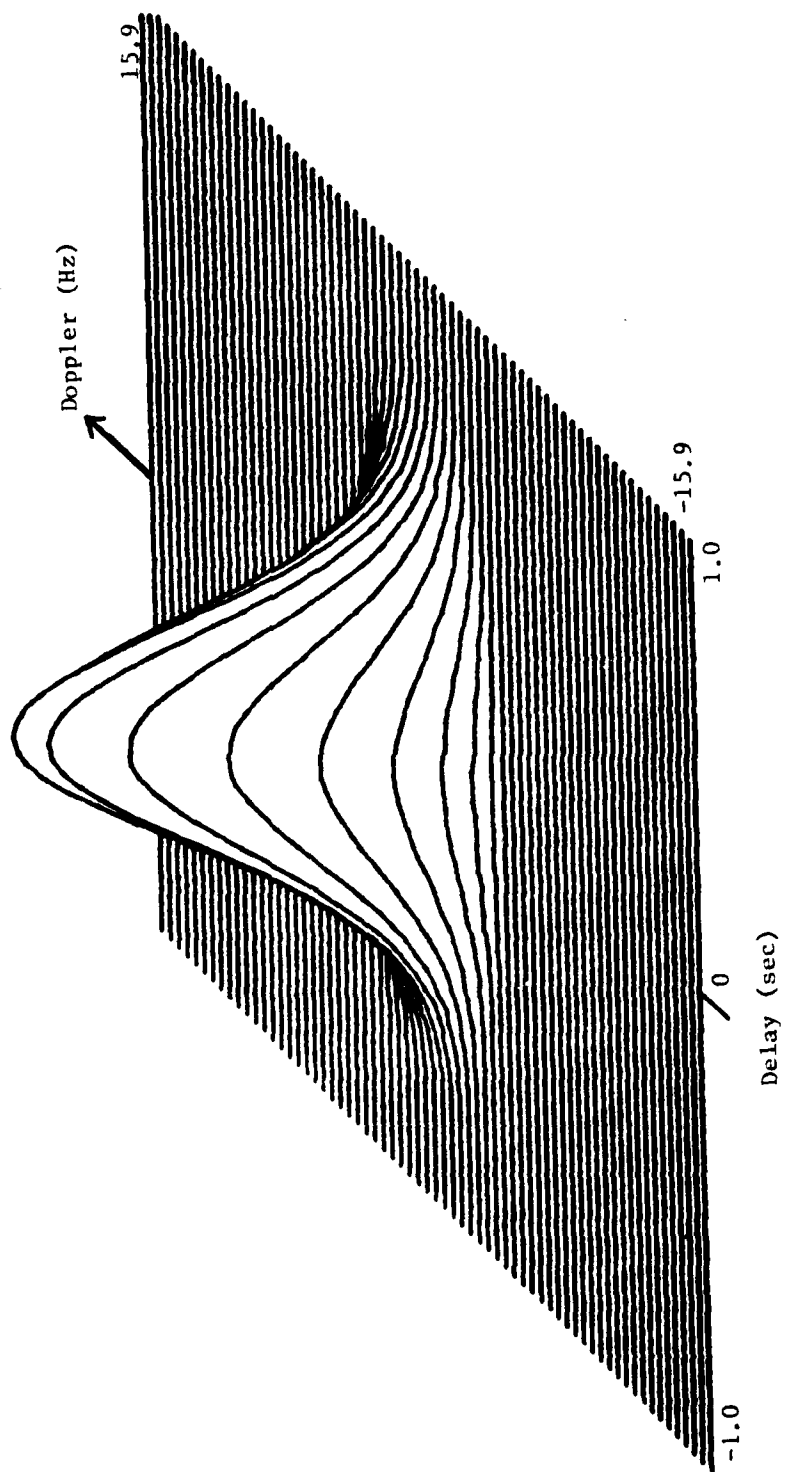
### Plots of the Magnitude of Ambiguity Functions



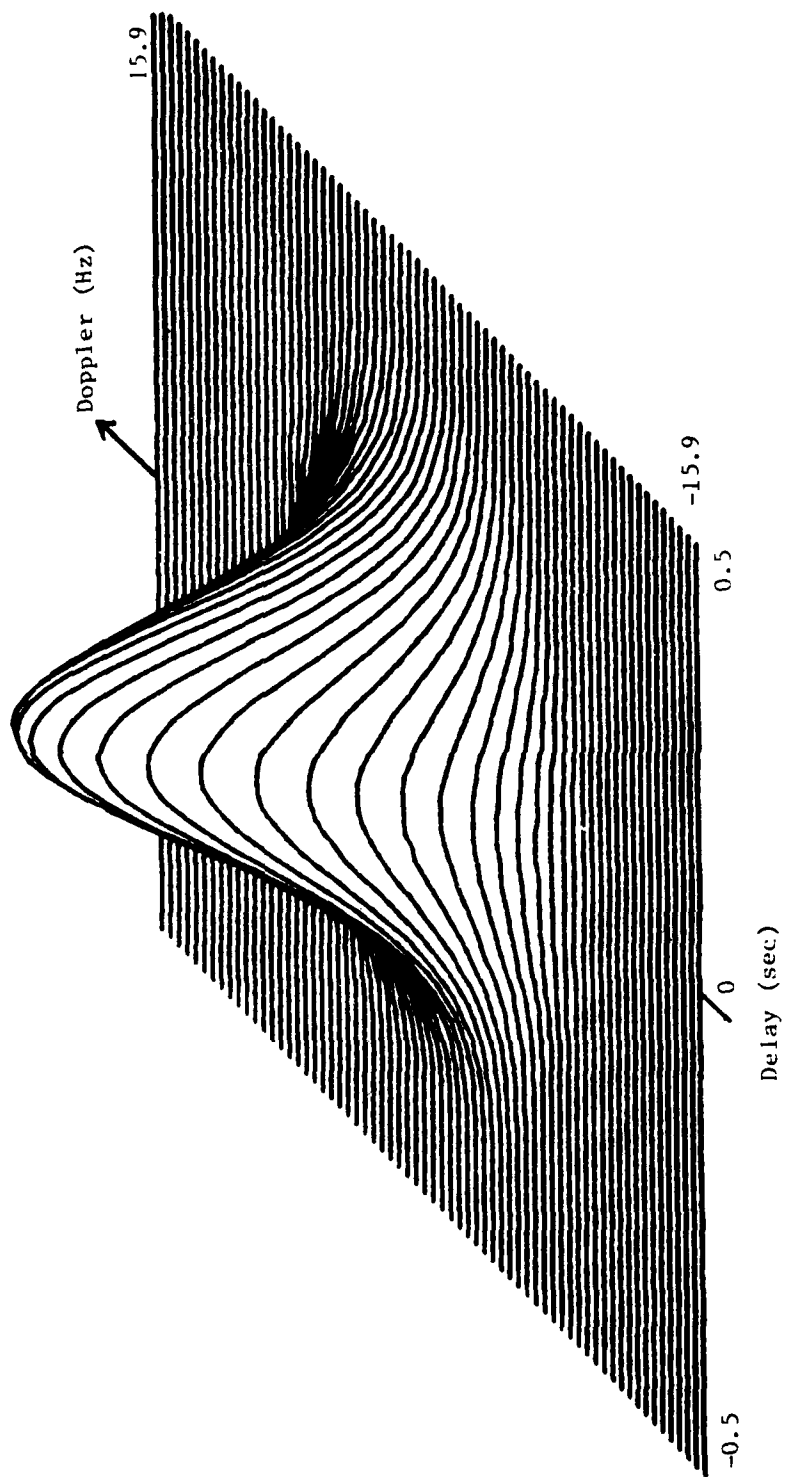
Rectangular Pulse  
PW=1.0 sec



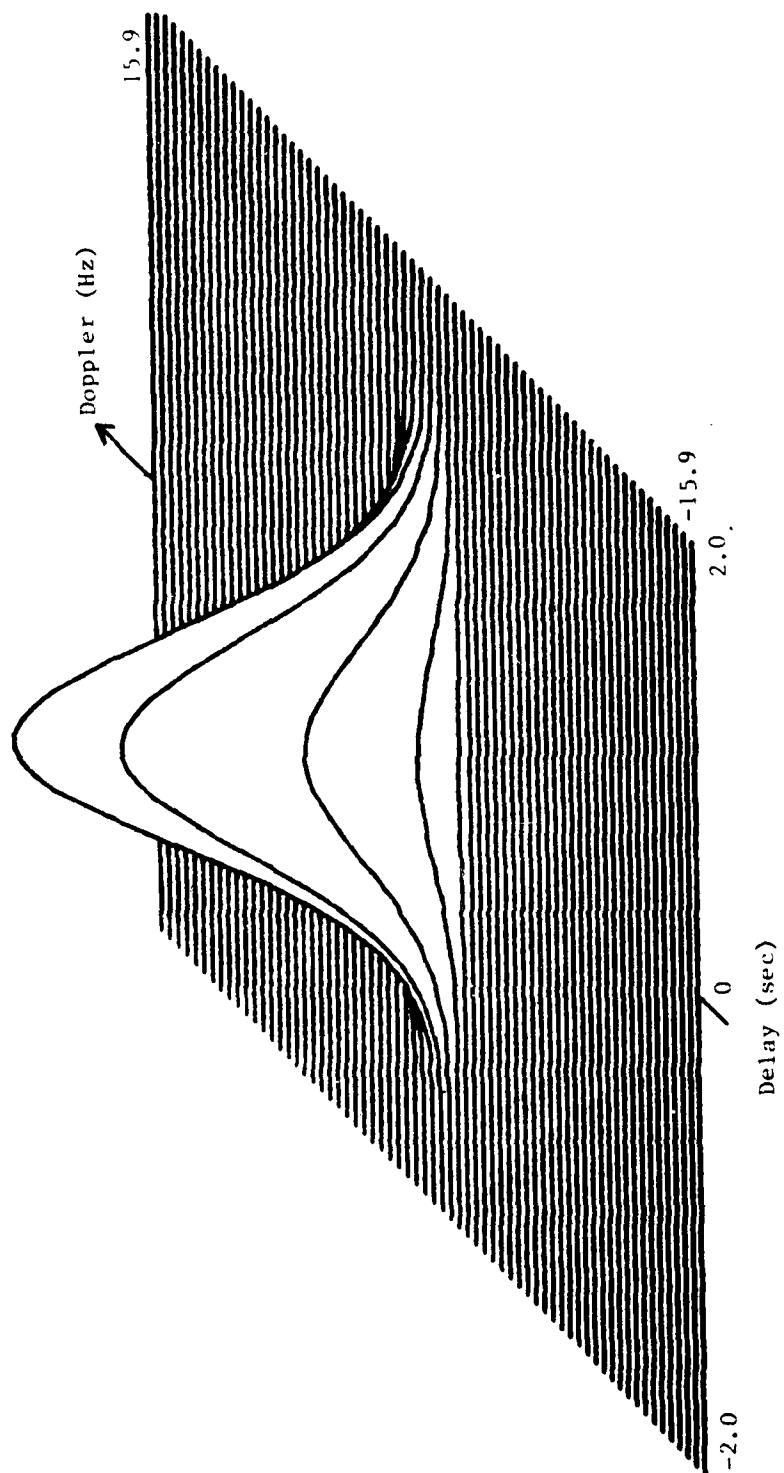
Gaussian Pulse  
PW=1.0 sec



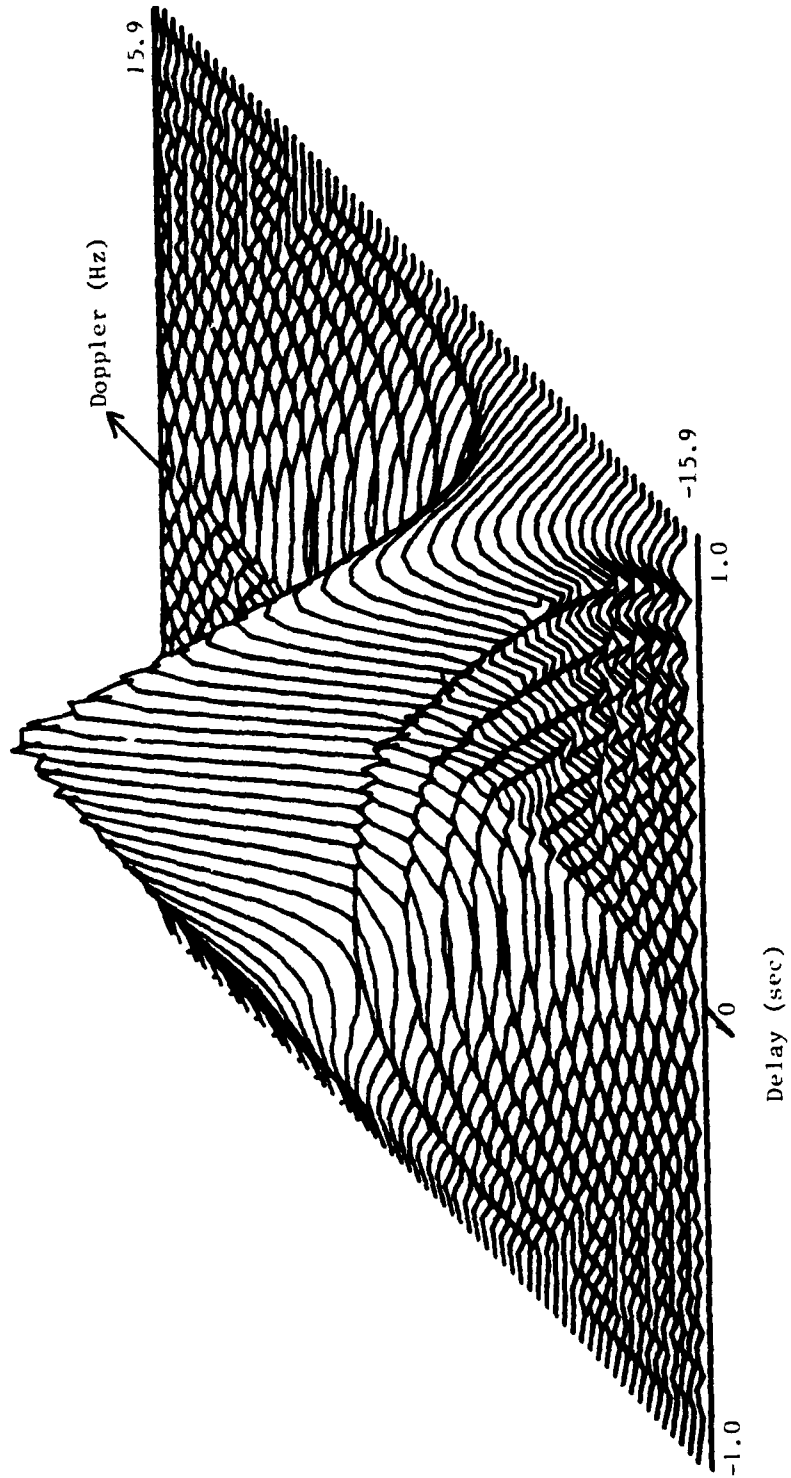
Gaussian Pulse  
PW=0.5 sec



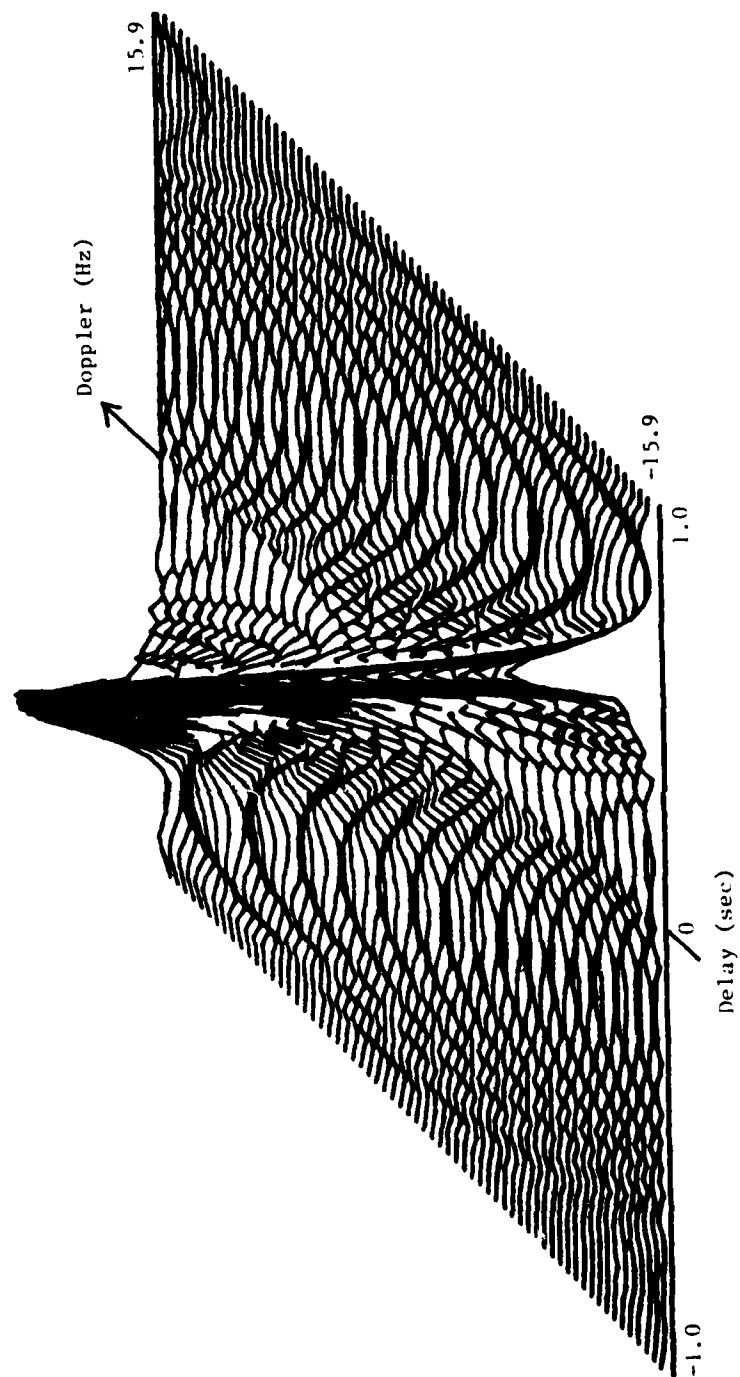
Gaussian Pulse  
PW=2.0 sec



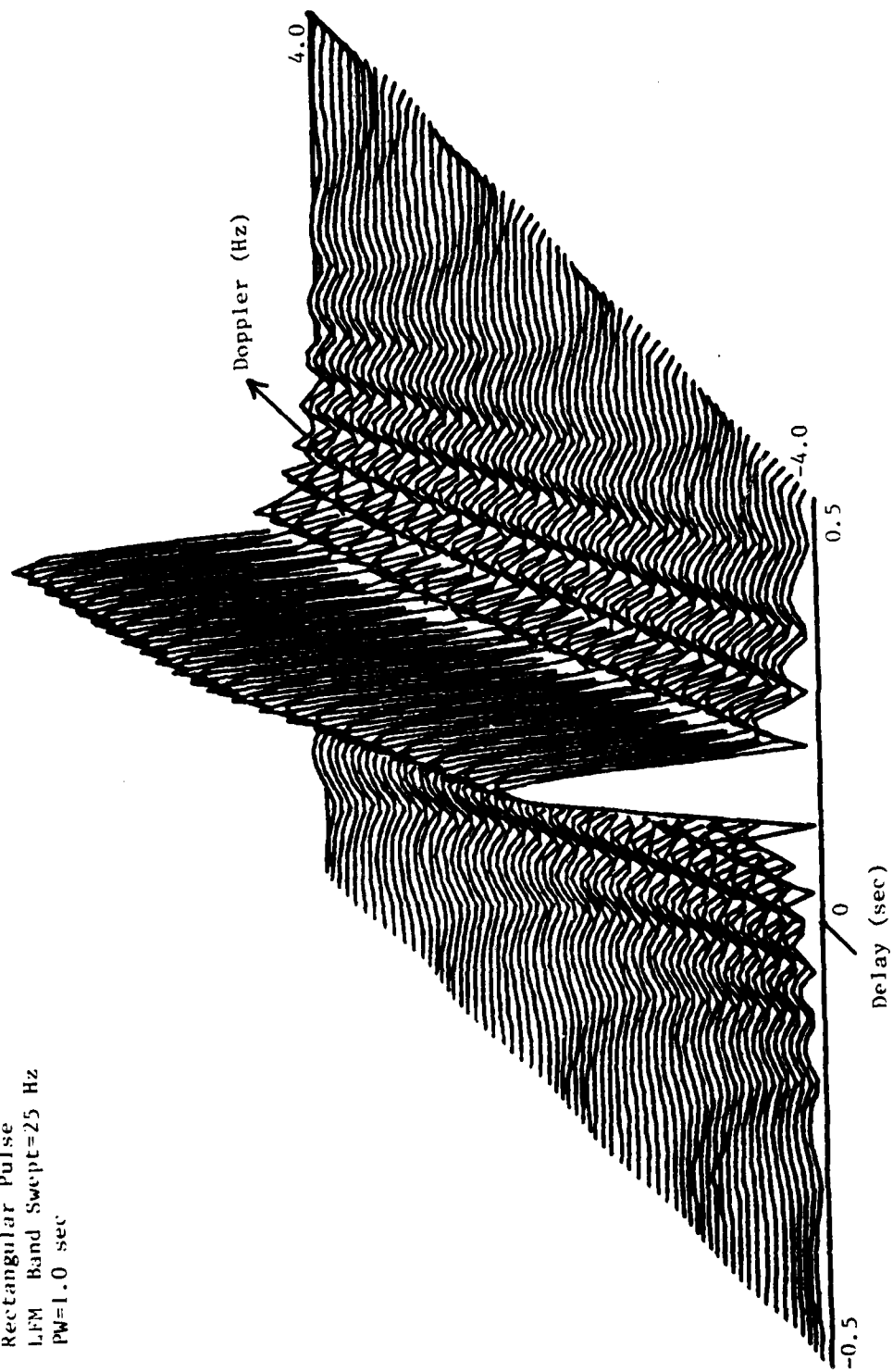
Rectangular Pulse  
LFM Band Swept=10 Hz  
PW=1.0 sec



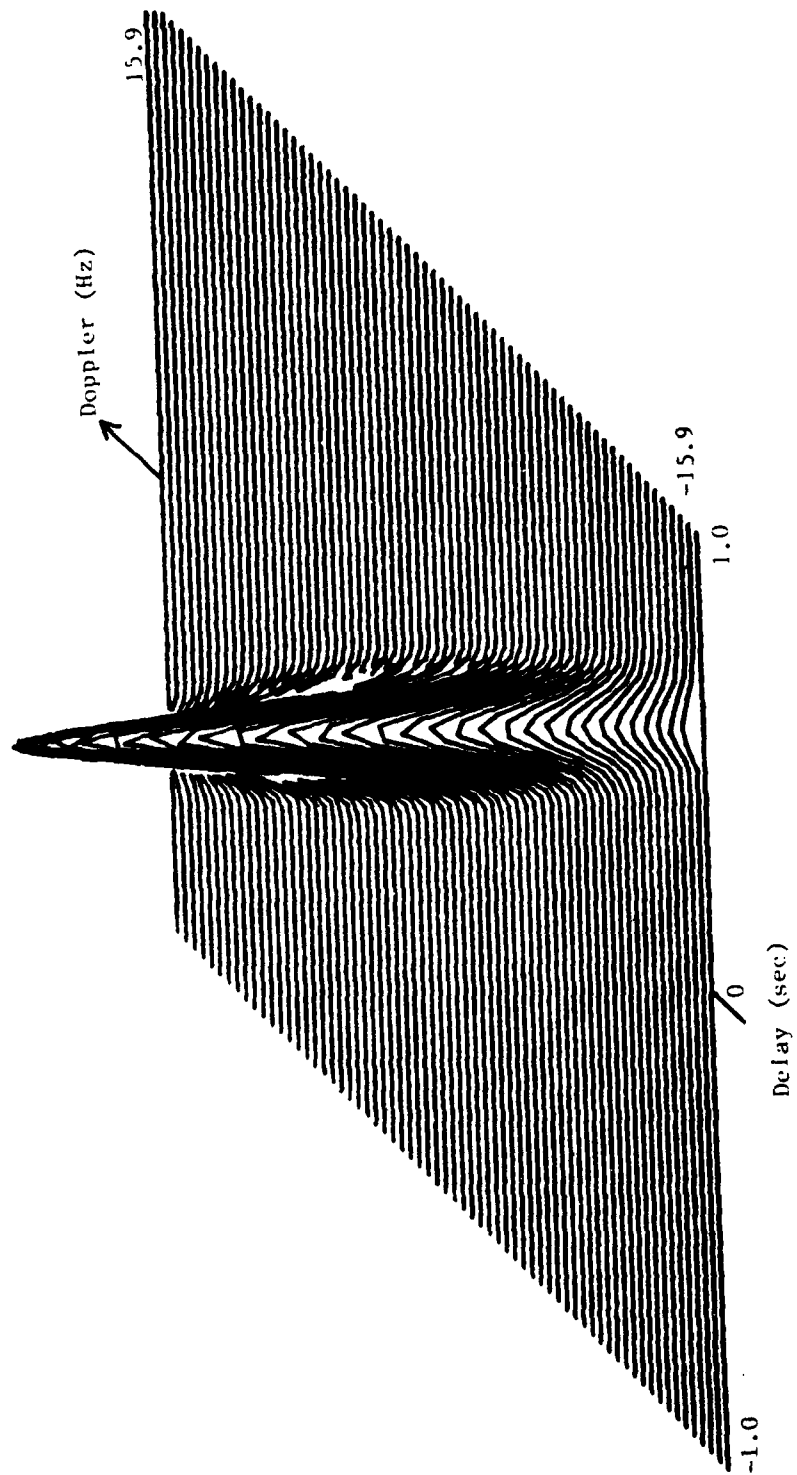
Rectangular Pulse  
LFM Band Swept=25 Hz  
PW=1.0 sec



Rectangular Pulse  
LFM Band Swept=25 Hz  
PW=1.0 sec

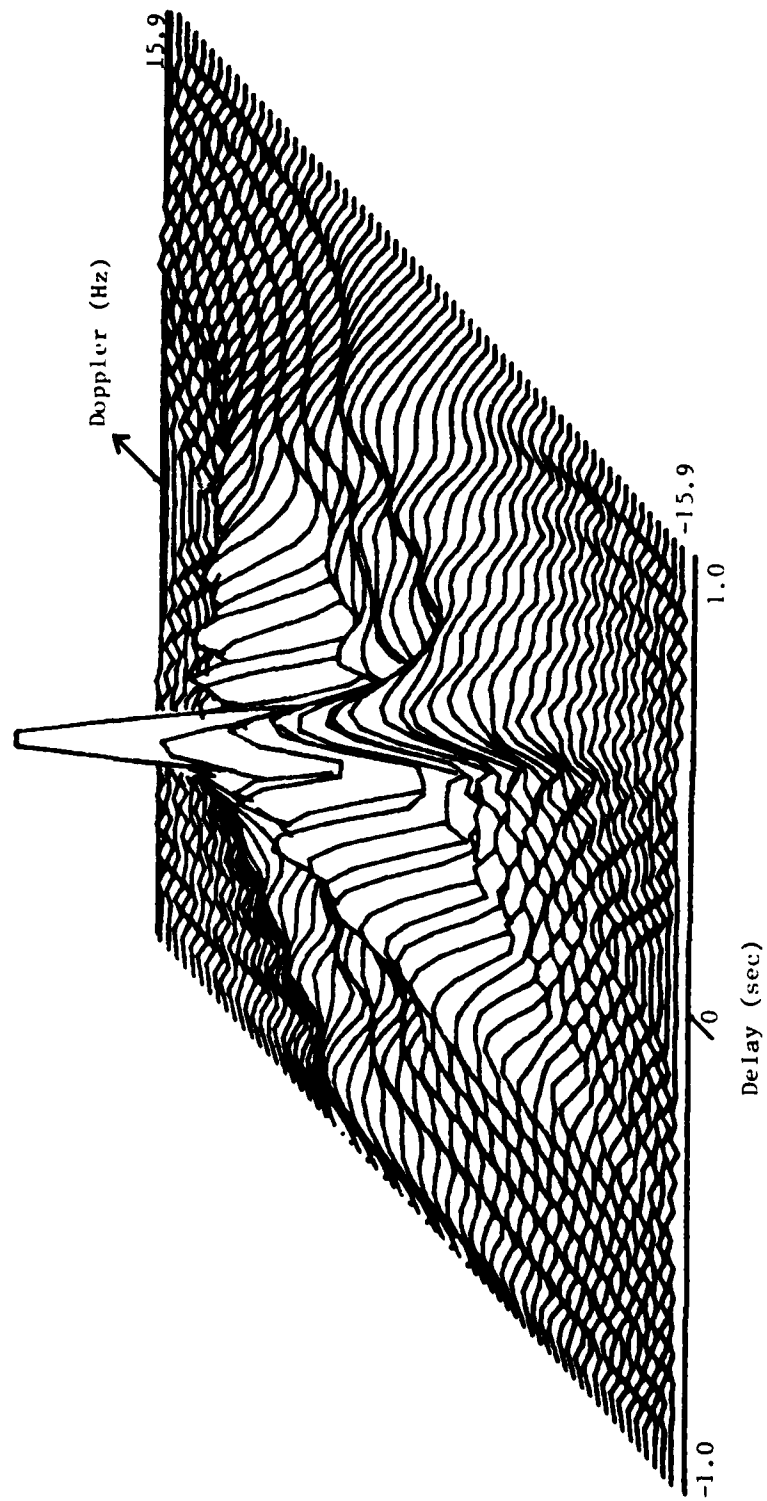


Gaussian Pulse  
LFM Band Swept=25 Hz  
PW=1.0 sec

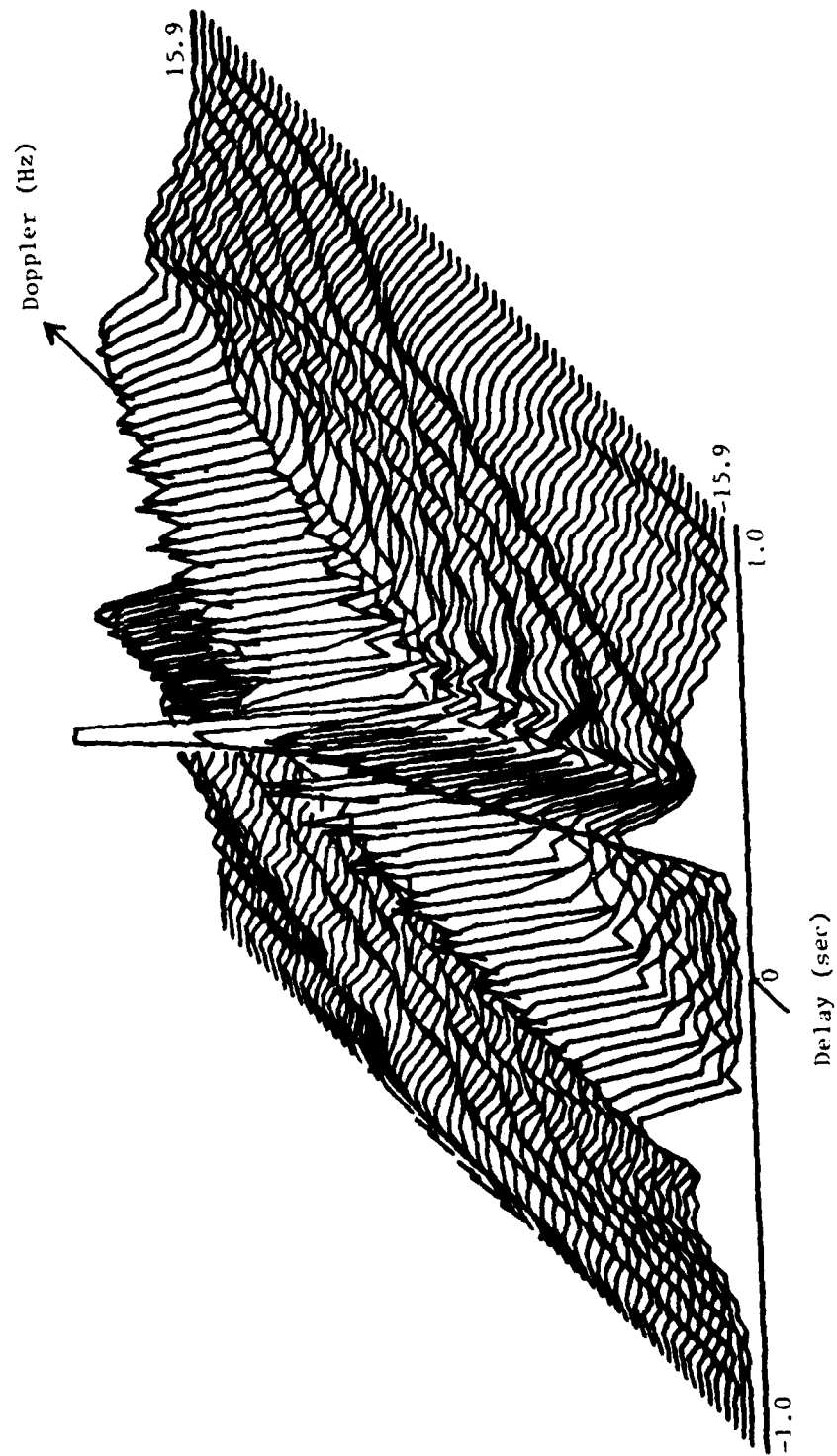




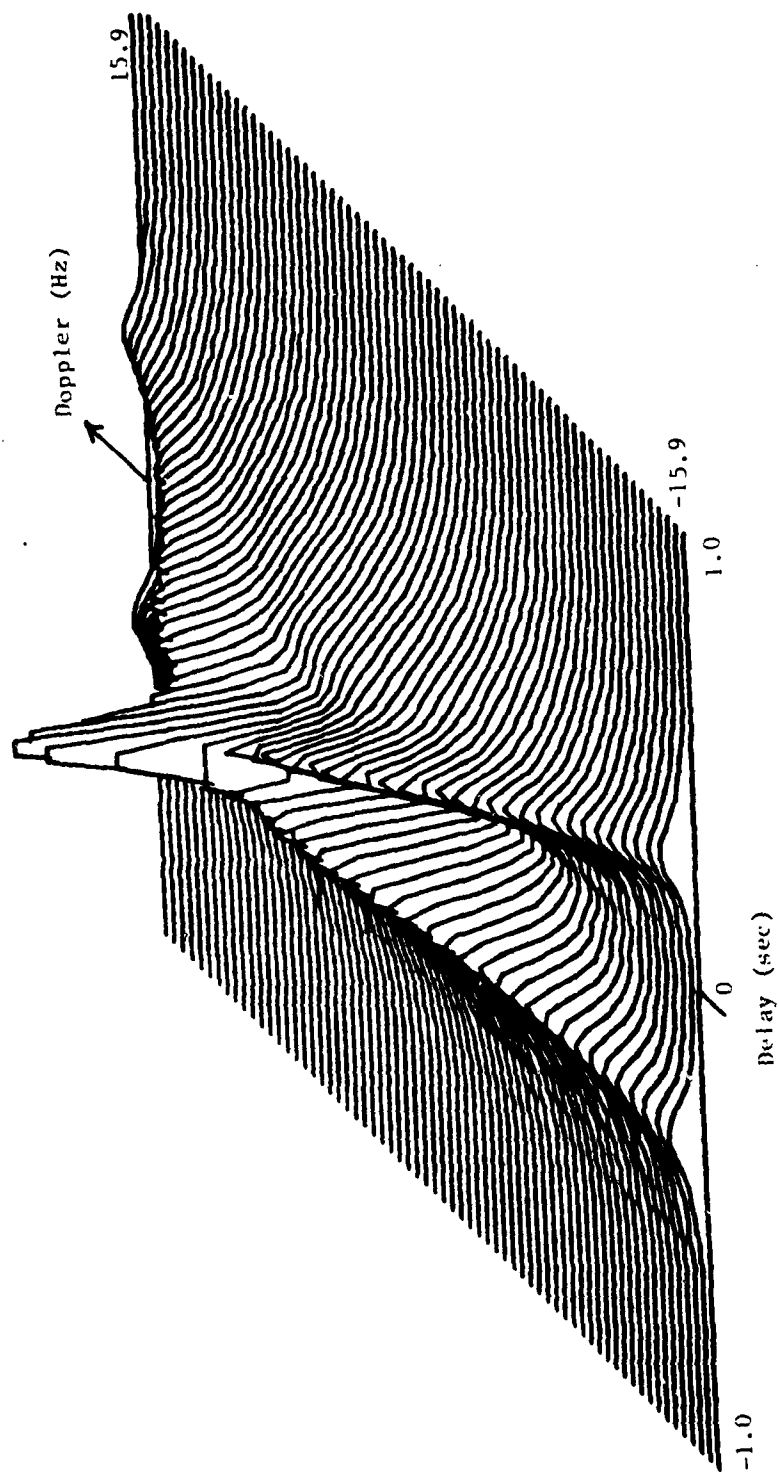
Rectangular Pulse  
VFM Band Swept=10 Hz  
PW=1.0 sec



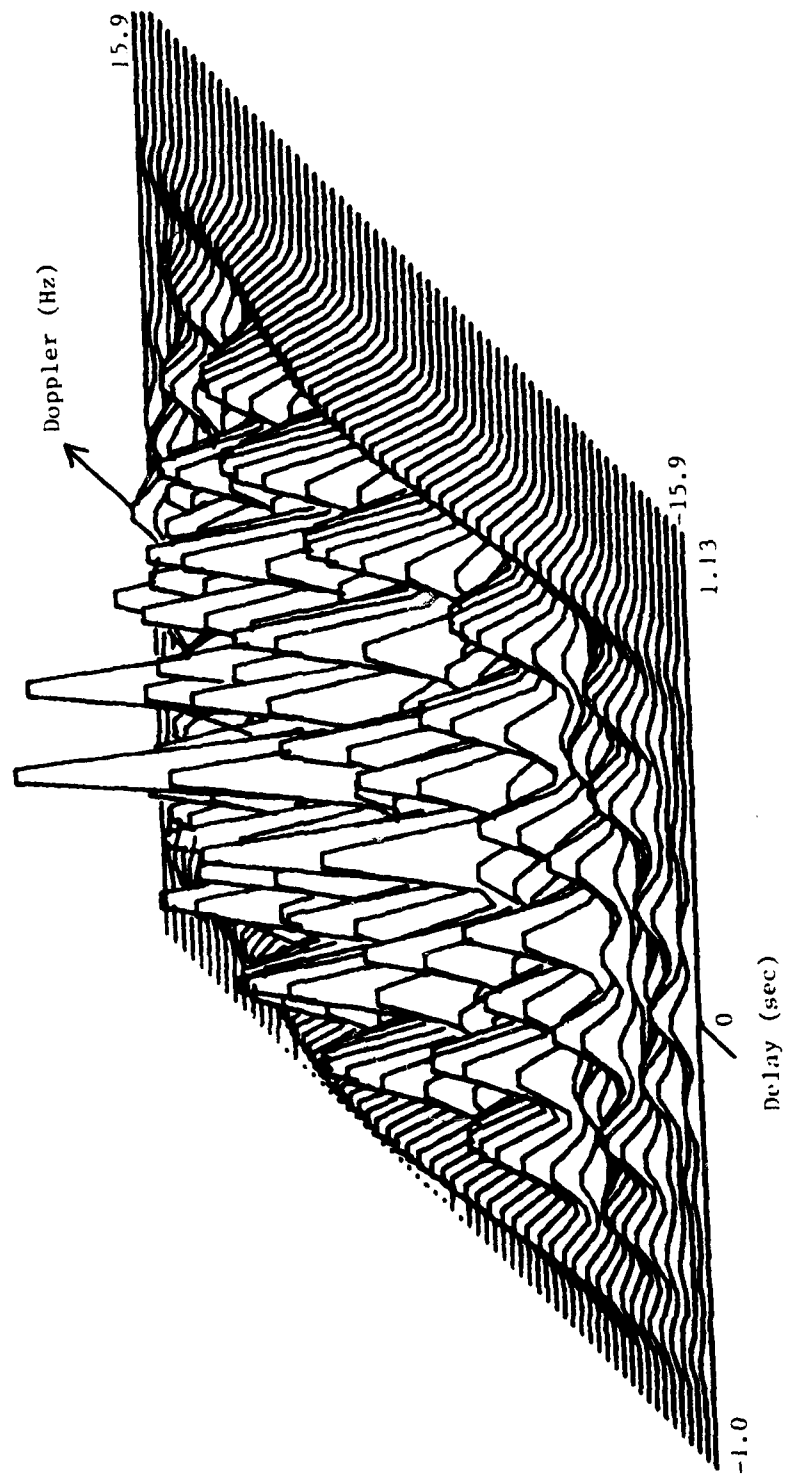
Rectangular Pulse  
VFM Band Swept=25 Hz  
PW=1.0 sec

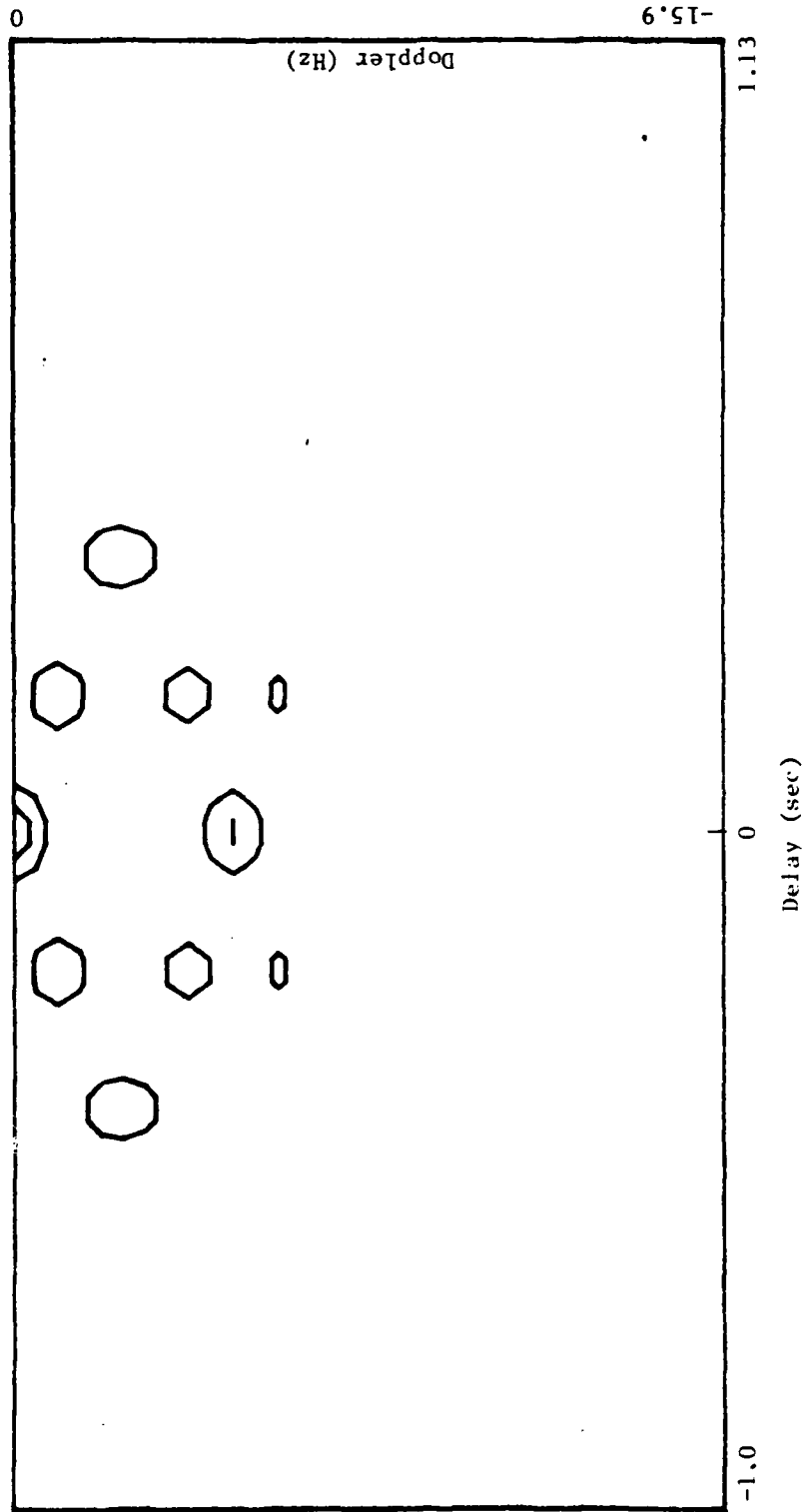


Gaussian Pulse  
VFM Band Swept=25 Hz  
PW=1.0 sec



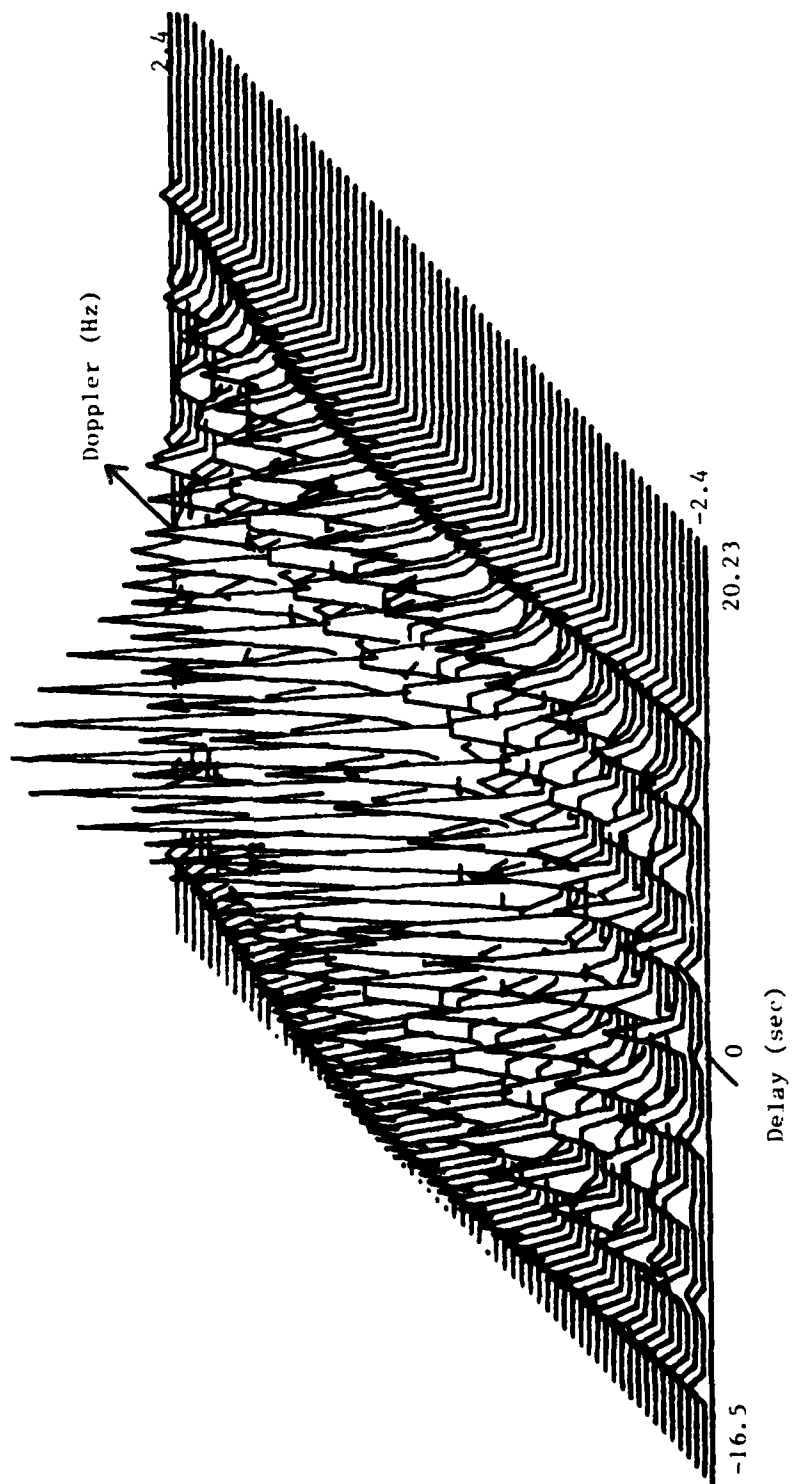
5-Element Barker Code  
Gaussian Elements  
PW=1.0 sec



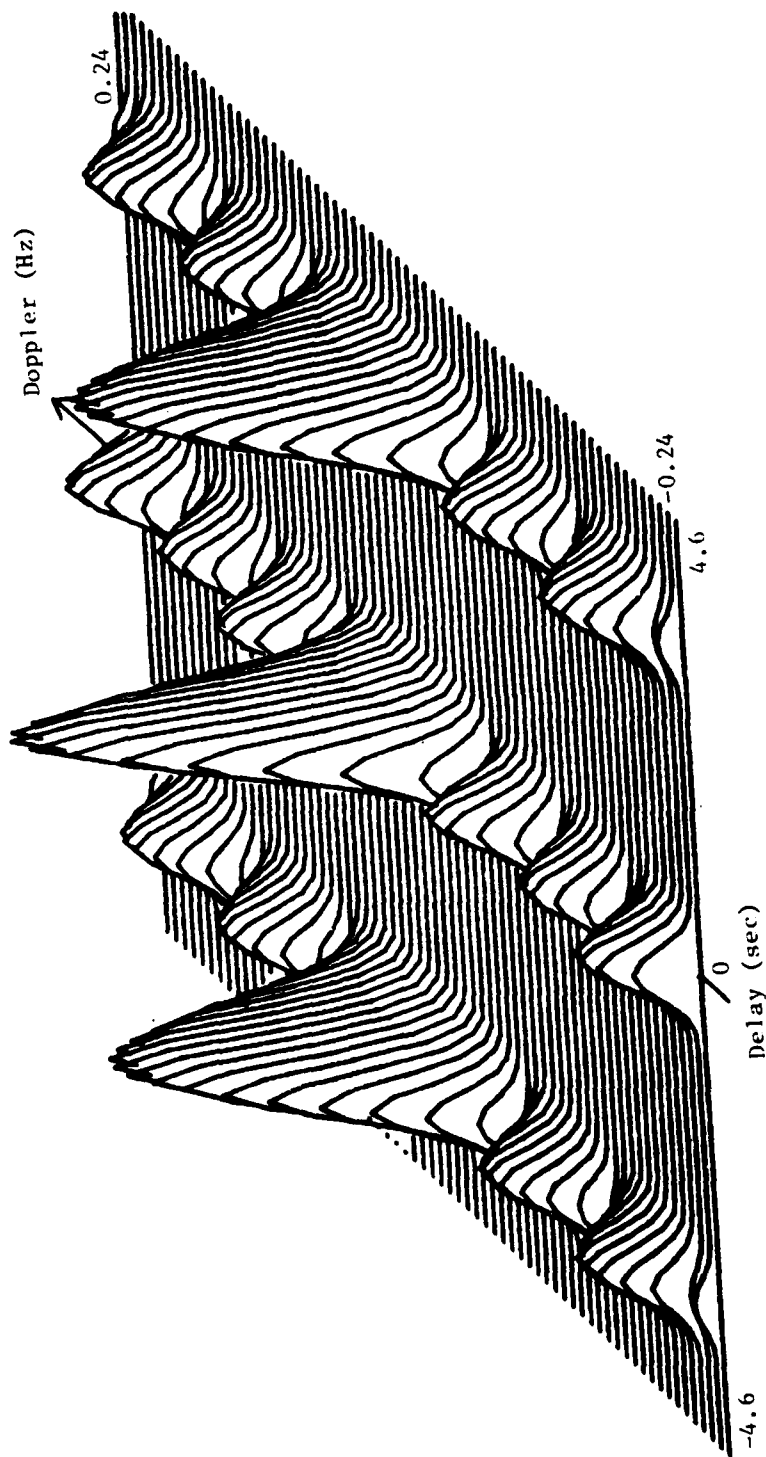


-3dB and -6dB Contour of 5-Element Barker Code

Repeated Gaussian Pulses  
5 Repetitions  
PW=1.0 sec PRT=3.3 sec



Central Region  
 Repeated Gaussian Pulses  
 5 Repetitions  
 PW=1.0 sec PRT=3.3 sec



APPENDIX D

Program Listings



```

C*****
C*      PROGRAM MAIN          *
C*      WRITTEN BY: CAPT JOHN REED    *
C*      15 APRIL 1982          *
C*****

```

```

C      MAIN IS A FORTRAN V PROGRAM WHICH CALCULATES THE MAGNITUDE OF A CUT OF
C      THE AMBIGUITY FCN FOR A SPECIFIED DOPPLER FREQUENCY. THE OUTPUT IS THE
C      MAGNITUDE AT VARIOUS DELAY TIMES (TAU) FOR THE SPECIFIED DOPPLER
C      FREQUENCY (W).

```

```

C      THE FOLLOWING ARE USER SELECTED PARAMETERS WHICH THE PROGRAM (OR
C      THE SUBROUTINES WHICH IT CALLS) WILL REQUEST:

```

- C 1. PULSE WIDTH.
- C 2. WHETHER OR NOT THE PULSE IS REPEATED.
  - C A. WHETHER PULSE REPETITION IS STAGGERED OR THE SAME.
  - C B. PULSE REPETITION TIME(S) (PRT)
  - C C. NUMBER OF REPETITIONS
- C 3. NUMBER OF SAMPLES TO BE MADE OF THE WAVEFORM.
- C 4. WHETHER OR NOT THE PULSE(S) IS PHASE-SHIFT CODED.
  - C A. NUMBER OF ELEMENTS IN THE PULSE CODE.
  - C B. THE PHASE OF EACH ELEMENT.
- C 5. WHETHER OR NOT THE PULSE(S) IS FREQUENCY MODULATED.
  - C A. BANDWIDTH TO BE SWEPT BY THE FM.
  - C B. IF LINEAR FM OR VFM.
- C 6. WHETHER THE PULSE(S) IS RECTANGULAR OR GAUSSIAN SHAPED.
- C 7. DOPPLER FREQUENCY, W (RADIAN), FOR WHICH THE MAGNITUDE OF
 C THE AMBIGUITY FUNCTION IS TO BE CALCULATED.

```

C      MAIN MAY MAKE CALLS TO THE FOLLOWING SUBROUTINES:

```

```

C      INPUT, PHASE, FM, GPULS, RECPL, SETUP, OUT.

```

```

C      IT ALSO CALLS THE FAST FOURIER TRANSFORM SUBROUTINE (EDFT.LB) FROM
C      THE COMPUTER LIBRARY.

```

```

C      MAIN IS COMPILED AND LOADED ON THE ECLIPSE USING THE FOLLOWING COMMANDS.

```

```

C      FORTRAN MAIN

```

```

C      RLDR/N MAIN INPUT PHASE FM GPULS RECPL SETUP OUT EDFT.LB @FLIB@

```

```

C      THIS PROGRAM IS STORED IN FILE 'MAIN.FR' IN THE DIRECTORY DP4:REED.

```

```

C      THE FOLLOWING VARIABLES ARE USED IN 'MAIN':

```

```

C      PW - PULSE WIDTH
C      SAME - PRT IF PULSES ARE REPEATED AT SAME INTERVAL
C      NPRT - # OF REPETITIONS OF PULSE TO BE MADE
C      PRT - ARRAY CONTAINING THE PRT FOR EACH REPETITION
C      TOTAL - TOTAL TIME DURATION OF RADAR WAVEFORM
C      NUM - # OF SAMPLES MADE OF WAVEFORM
C      SAMPLE- # OF SAMPLES OF WAVEFORM PLUS ZERO-FILLING SAMPLES NEEDED
C              TO PERFORM LINEAR CONVOLUTION (=2*NUM)
C      PERIOD- ARRAY CONTAINING # OF SAMPLES MADE DURING EACH PRT
C      N - # OF SAMPLES MADE OF PULSE AFTER WEIGHTING
C      M - # OF NON-ZERO-FILL SAMPLES MADE OF WAVEFORM AFTER WEIGHTING
C      Z - ARRAY CONTAINING THE AMPLITUDE OF EACH SAMPLE
C      CFACOR-SCALING VALUE OF MAGNITUDE
C      FREQ - ARRAY CONTAINING FREQUENCY MODULATION FOR EACH SAMPLE

```

```

C      SCALE - SCALING FACTOR OF MAGNITUDE
C      VALUE - COMPLEX EXPONENTIAL OF DOPPLER FREQUENCY (EXP(J*W))
C      XVALS - ARRAY CONTAINING THE TIME-REVERSED, CONJUGATE SAMPLES OF
C              THE COMPLEX ENVELOPE OF THE WAVEFORM; ALSO CONTAINS THE
C              AMBIGUITY FUNCTION VALUES LATER IN THE PROGRAM
C      YVALS - ARRAY CONTAINING THE DOPPLER AND RANGE-DELAYED SAMPLES
C              OF THE COMPLEX ENVELOPE OF WAVEFORM
C      MAG - ARRAY CONTAINING THE MAGNITUDE OF THE AMBIGUITY FCN CUT
C              FOR EACH SAMPLE

COMPLEX XVALS(1024), YVALS(1024), Z(1024), VALUE
INTEGER SAMPLE, PERIOD(10)
REAL MAG(1024), PRT(10), FREQ(1024)

TYPE '          WELCOME TO MAIN'
TYPE 'THIS PROGRAM IS DESIGNED TO COMPUTE THE MAGNITUDE OF THE'
TYPE 'AMBIGUITY FUNCTION OF A RADAR WAVEFORM. YOU WILL BE ASKED'
TYPE 'TO ENTER PARAMETERS TO DEFINE THE WAVEFORM AT VARIOUS'
TYPE 'TIMES DURING THE PROGRAM. WHEN THE PROGRAM REQUESTS AN'
TYPE 'ENTRY, SIMPLY TYPE YOUR REPLY AND, IF CORRECT, HIT CR KEY.'
TYPE 'IF YOU MAKE A MISTAKE ON AN ENTRY, USE THE <DEL> KEY TO'
TYPE 'ERASE THE CHARACTER OR THE <\> KEY TO DELETE THE ENTIRE'
TYPE 'ENTRY. YOU MAY STOP THE PROGRAM AT ANY TIME BY ENTERING'
TYPE '<QA> THE OUTPUT WILL BE THE MAGNITUDE OF THE AMBIGUITY'
TYPE 'FUNCTION AT VARIOUS DELAY TIMES (TAU) FOR THE DOPPLER FREQUENCY'
TYPE 'SPECIFIED EARLIER. TO STOP THE SCREEN DISPLAY OUTPUT, ENTER'
TYPE '<OS> TO RESUME THE DISPLAY OUTPUT, ENTER '<OQ>.'
PAUSE 'ENTER ANY CHARACTER TO BEGIN'
CALL INPUT(PERIOD, PRT, PW, NPRT, TOTAL, SAMPLE, NUM, M, N)
ACCEPT 'ENTER 0 IF PULSE(S) IS PHASE-SHIFT CODED, 1 IF NOT ', J
IF (J EQ 1) GO TO 5
CALL PHASE(FREQ, PRT, PERIOD, Z, M, N, NPRT, GFACTOR, PW, SAMPLE, TOTAL)
GO TO 20
5 TYPE 'PULSE(S) IS NOT PHASE-SHIFT CODED'
ACCEPT 'ENTER 0 IF PULSE(S) IS FREQUENCY MODULATED, 1 IF NOT ', J
CALL FM(FREQ, PERIOD, NPRT, PW, N, J, SAMPLE)
10 ACCEPT 'ENTER 0 FOR RECTANGULAR-SHAPED PULSE(S), 1 FOR GAUSSIAN ', J
IF (J EQ 0) GO TO 15
CALL GPULS(PERIOD, Z, PW, GFACTOR, NPRT, N)
GO TO 20
15 CALL RECPL(PERIOD, Z, GFACTOR, NPRT, N)
20 SCALE=GFACTOR/(N*NPRT)
ACCEPT 'ENTER VALUE OF W TO BE CONSIDERED ', W
25 VALUE=CMPLX(0.0, W)
WRITE (10, 300) W
CALL SETUP(Z, FREQ, XVALS, YVALS, VALUE, TOTAL, SAMPLE, M)
CALL DFT5(XVALS, SAMPLE, 0)
CALL DFT5(YVALS, SAMPLE, 0)
DO 30 I=1, SAMPLE
XVALS(I)=CONJG(XVALS(I))
YVALS(I)=CONJG(YVALS(I))
XVALS(I)=XVALS(I)*YVALS(I)/SAMPLE
30 CONTINUE
CALL DFT5(XVALS, SAMPLE, 1)

```

```

DO 35 I=1, SAMPLE
MAG(I)=CABS(XVALS(I))*SCALE
35  CONTINUE
CALL OUT(MAG, NUM, PW, TOTAL, M)
ACCEPT 'ENTER 999 TO STOP OR NEXT W TO CONTINUE ', W
IF (W EQ 999) GO TO 500
GO TO 25
300  FORMAT('W=', F12.2)
500  END

```

```

C*****
C*      SUBROUTINE INPUT      *
C*      WRITTEN BY  CAPT JOHN REED  *
C*      15 APRIL 1982      *
C*****

C      INPUT IS A FORTRAN V SUBROUTINE WHICH REQUESTS INFORMATION FROM
C      THE USER TO DEFINE THE BASIC RADAR WAVEFORM
C
C      THE INPUT SUBROUTINE IS CALLED BY THE FOLLOWING PROGRAMS
C      MAIN, GMAIN, IMAIN.
C      THIS SUBROUTINE RECEIVES NO VARIABLES FROM THE CALLING PROGRAM
C      IT RETURNS THE FOLLOWING VARIABLES TO THE CALLING PROGRAM:
C      PERIOD, PRT, PW, NPRT, TOTAL, SAMPLE, NUM, M, N.
C
C      INPUT IS COMPILED ON THE ECLIPSE USING THE COMMAND:
C      FORTRAN INPUT
C
C      THIS SUBROUTINE IS STORED IN FILE 'INPUT.FR' IN THE DIRECTORY DP4 REED
C
C      THE FOLLOWING VARIABLES ARE USED IN THIS SUBROUTINE.
C      PW - PULSE WIDTH
C      SAME - PRT IF PULSES ARE REPEATED AT THE SAME INTERVAL
C      NPRT - # OF REPETITIONS OF PULSE TO BE MADE
C      PRT - ARRAY CONTAINING THE PRT FOR EACH REPETITION
C      TOTAL - TOTAL DURATION OF WAVEFORM (IN TIME)
C      NUM - # OF SAMPLES TO BE MADE OF WAVEFORM
C      SAMPLE- # OF SAMPLES OF WAVEFORM PLUS THE ZERO-FILL SAMPLES
C              NEEDED TO PERFORM THE LINEAR CONVOLUTION (=2*NUM)
C      N - # OF SAMPLES MADE OF EACH PULSE AFTER PROPER WEIGHTING
C      M - # OF NON-ZERO-FILL SAMPLES MADE OF WAVEFORM AFTER WEIGHTING
C      PERIOD- ARRAY CONTAINING # OF SAMPLES MADE OF EACH PRT

      SUBROUTINE INPUT(PERIOD, PRT, PW, NPRT, TOTAL, SAMPLE, NUM, M, N)

      INTEGER SAMPLE, PERIOD(10)
      REAL PRT(10)

      ACCEPT 'ENTER PULSE WIDTH ', PW
      ACCEPT 'ENTER 1 IF WAVEFORM HAS REPEATED PULSES, 0 IF NOT ', I
      IF (I EQ. 0) GO TO 25
      TYPE 'WAVEFORM HAS REPEATED PULSES'
      ACCEPT 'ENTER 1 IF STAGGERED PRT, 0 IF PRT SAME ', J
      IF (J EQ. 1) GO TO 10
      ACCEPT 'ENTER PULSE REPITITION TIME ', SAME
      ACCEPT 'ENTER # OF REPITITIONS TO BE MADE ', NPRT
      DO 5 I=1, NPRT
      PRT(I)=SAME
5      CONTINUE
      TOTAL=SAME*NPRT
      WRITE(10, 300) SAME, NPRT
      GO TO 30
10     TYPE "STAGGERED PRT'S TO BE USED"
      ACCEPT 'ENTER # OF DIFFERENT PRT TO BE USED ', NPRT

```

```

ACCEPT 'ENTER FIRST PRT ',PRT(1)
DO 15 I=2,NPRT
ACCEPT 'ENTER NEXT PRT ',PRT(I)
15 CONTINUE
TOTAL=0.0
DO 20 I=1,NPRT
TOTAL=TOTAL+PRT(I)
20 WRITE(10,305) I,PRT(I)
CONTINUE
GO TO 30
25 TYPE 'WAVEFORM HAS ONLY ONE PULSE'
TOTAL=PW
NPRT=1
PRT(1)=PW
ACCEPT 'ENTER # OF SAMPLES TO BE MADE OF WAVEFORM ',NUM
SAMPLE=NUM*2
PERIOD(1)=NUM
M=NUM
N=NUM
TYPE 'N=',N,' M=',M,' PERIOD(1)=' ,PERIOD(1)
GO TO 33
30 TYPE 'ENTER # OF SAMPLES TO BE MADE OF WAVEFORM; YOU MAY'
ACCEPT ' CHOOSE 16,32,64,128,256,OR 512 ',NUM
SAMPLE=NUM*2
IF (PW GT 0.0001) GO TO 32
CPW=PW*10000.0
CTOTAL=TOTAL*10000.0
N=NUM/CTOTAL*CPW
M=N*CTOTAL/CPW
DO 31 I=1,NPRT
CPRT(I)=PRT(I)*10000.0
PERIOD(I)=N*CPRT(I)/CPW
TYPE 'N=',N,' M=',M
TYPE 'I=',I,' PERIOD(I)=' ,PERIOD(I)
31 CONTINUE
GO TO 33
32 N=NUM/TOTAL*PW
M=N*TOTAL/PW
TYPE 'M=',M
TYPE 'N=',N
DO 35 I=1,NPRT
PERIOD(I)=N*PRT(I)/PW
TYPE 'I=',I,' PERIOD(I)=' ,PERIOD(I)
35 CONTINUE
33 RETURN
300 FORMAT('PRT'S ARE THE SAME...PRT=',F12.10,5X,'# OF REPITITIONS=',I2)
305 FORMAT('FOR I=',I,5X,'PRT=',F12.10)
END

```

```

C*****
C*      SUBROUTINE PHASE      *
C*      WRITTEN BY: CAPT JOHN REED      *
C*      15 APRIL 1982      *
C*****

C      PHASE IS A FORTRAN V SUBROUTINE WHICH CALCULATES THE AMPLITUDE OF
C      THE SAMPLES OF A PULSE WHICH IS PHASE-SHIFT CODED. IT ALSO SETS THE
C      FREQUENCY OF EACH SAMPLE TO ZERO SINCE FM IS NOT AN ALLOWED OPTION.
C
C      THE PHASE SUBROUTINE MAY BE CALLED BY THE FOLLOWING PROGRAMS:
C      MAIN, GMAIN, IMAIN.
C
C      THIS SUBROUTINE RECEIVES THE FOLLOWING VARIABLES FROM THE CALLING PROGRAM:
C      PRT, PERIOD, M, N, NPRT, PW, SAMPLE, TOTAL.
C      IT RETURNS THE FOLLOWING VARIABLES TO THE CALLING PROGRAM:
C      FREQ, PRT, PERIOD, Z, M, N, GFACTOR, TOTAL.
C
C      'PHASE' IS COMPILED ON THE ECLIPSE USING THE COMMAND:
C      FORTRAN PHASE
C      'PHASE' IS STORED IN FILE 'PHASE.FR' IN THE DIRECTORY DP4: REED.
C
C      THE FOLLOWING VARIABLES ARE USED IN SUBROUTINE PHASE:
C      NELEM - # OF ELEMENTS IN THE PULSE CODE
C      ITIME - # OF SAMPLES TO BE MADE OF EACH ELEMENT
C      SUM - TOTAL # OF SAMPLES MADE OF EACH PULSE AFTER WEIGHTING
C      PERIOD- ARRAY CONTAINING # OF SAMPLES MADE OF EACH PRT AFTER WEIGHTING
C      RATIO - ARRAY CONTAINING THE PHASE OF EACH ELEMENT (/PI)
C      MEAN - 1/2 LENGTH OF AN ELEMENT
C      STDEV - 1/3 OF MEAN; 3 STANDARD DEVIATIONS FROM MEAN
C      VAR - VARIANCE (STDEV*STDEV)
C      GAUSS - SCALING VALUE OF GAUSSIAN ELEMENTS (1/SQRT(2*PI*VAR))
C      GFACTOR-SCALING VALUE OF MAGNITUDE OF AMBIGUITY FCN
C      TIME - VALUE OF DELAY (TAU)
C      POWER - (TIME-MEAN)
C      WT - VALUE OF GAUSSIAN PULSE (DISTRIBUTION) FOR A PARTICULAR TIME
C      AMT2 - IMAGINARY VALUE = RATIO*PI
C      Z - ARRAY CONTAINING AMPLITUDE OF EACH SAMPLE OF WAVEFORM
C      FREQ - ARRAY CONTAINING THE FM OF EACH SAMPLE
C      M - TOTAL # OF NON-ZERO-FILL SAMPLES OF WAVEFORM AFTER WEIGHTING
C      N - # OF SAMPLES OF EACH ELEMENT AFTER WEIGHTING

SUBROUTINE PHASE(FREQ, PRT, PERIOD, Z, M, N, NPRT, GFACTOR, PW, SAMPLE, TOTAL)

COMPLEX Z(1024), AMT2
INTEGER PERIOD(10), COUNT, SUM, TALLY, SAMPLE
REAL PRT(10), RATIO(20), FREQ(1024), MEAN

TYPE 'PULSES ARE PHASE-SHIFT CODED. ... NO OTHER OPTIONS AVAILABLE'
ACCEPT 'ENTER # ELEMENTS IN THE PULSE CODE ', NELEM
ITIME=N/NELEM
SUM=ITIME*NELEM
TYPE 'SUM=', SUM
IF (NPRT.NE.1) GO TO 1

```

```

PERIOD(1)=SUM
TYPE 'PERIOD(1)=',PERIOD(1)
GO TO 6
1 IF (PW GT 0.0001) GO TO 3
DO 2 I=1,NPRT
CPRT(I)=PRT(I)*10000.0
CPW=PW*10000.0
PERIOD(I)=SUM*CPRT(I)/CPW
TYPE 'I=',I,' PERIOD(I)=',PERIOD(I)
2 CONTINUE
GO TO 6
3 DO 5 I=1,NPRT
PERIOD(I)=SUM*PRT(I)/PW
TYPE 'I=',I,' PERIOD(I)=',PERIOD(I)
5 CONTINUE
6 DO 10 I=1,NELEM
ACCEPT 'ENTER PHASE AS A*PI/B; ENTER A/B RATIO ',RATIO(I)
WRITE(10,300) I,RATIO(I)
10 CONTINUE
ACCEPT 'ENTER 0 FOR RECTANGULAR-SHAPED ELEMENTS, 1 FOR GAUSSIAN ',J
IF (J.EQ.0) GO TO 35
TYPE 'ELEMENTS ARE GAUSSIAN SHAPED'
MEAN=PW/(2.0*NELEM)
STDEV=MEAN/3.0
VAR=STDEV*STDEV
GAUSS=1.0/SGRT(2.0*VAR*3.1416)
GFACTOR=0.5923581*(PW/NELEM)*(PW/NELEM)
WRITE(10,305) MEAN,STDEV,VAR,GFACTOR
TALLY=0
DO 30 IA=1,NPRT
COUNT=TALLY+1
DO 20 I=1,NELEM
DO 15 J=COUNT,COUNT+ITIME-1
TIME=(PW/ITIME)*(J-COUNT+1)/NELEM
POWER=TIME-MEAN
WT=GAUSS*EXP(-POWER*POWER/(2.0*VAR))
AMT2=CMPLX(0.0,RATIO(I)*3.1416)
Z(J)=WT*CEXP(AMT2)
15 CONTINUE
COUNT=COUNT+ITIME
20 CONTINUE
DO 25 J=COUNT,COUNT+PERIOD(IA)-SUM-1
Z(J)=(0.0,0.0)
25 CONTINUE
TALLY=TALLY+PERIOD(IA)
30 CONTINUE
GO TO 60
35 TYPE 'ELEMENTS ARE RECTANGULAR SHAPED'
GFACTOR=1.0
TALLY=0
DO 55 IA=1,NPRT
COUNT=TALLY+1
DO 45 I=1,NELEM
DO 40 J=COUNT,COUNT+ITIME-1

```

```

      AMT2=CMPLX(0.0,RATIO(I)*3.1416)
      Z(J)=1.0*CEXP(AMT2)
40    CONTINUE
      COUNT=COUNT+ITIME
45    CONTINUE
      DO 50 J=COUNT, COUNT+PERIOD(IA)-SUM-1
      Z(J)=(0.0,0.0)
50    CONTINUE
      TALLY=TALLY+PERIOD(IA)
55    CONTINUE
      DO 65 I=1, SAMPLE
      FREQ(I)=0.0
65    CONTINUE
      M=SUM*TOTAL/PW
      N=SUM
      RETURN
300   FORMAT('FOR I=', I2, 5X, 'PHASE=', F5.2, '*PI')
305   FORMAT('M=', F12.10, 3X, 'SD=', F12.10, 3X, 'VAR=', F12.10, 3X, 'GFAC=', F12.10)
      END

```



```

C*****
C*      SUBROUTINE FM      *
C*      WRITTEN BY: CAPT JOHN REED      *
C*      15 APRIL 1982      *
C*****

C      'FM' IS A FORTRAN V SUBROUTINE WHICH DETERMINES THE FREQUENCY
C      MODULATION OF EACH SAMPLE OF THE RADAR WAVEFORM (EXCEPT FOR
C      PHASE-SHIFT CODED SIGNALS).

C      THE FM SUBROUTINE MAY BE CALLED BY THE FOLLOWING PROGRAMS:
C      MAIN, QMAIN, IMAIN.

C      THIS SUBROUTINE RECEIVES THE FOLLOWING VARIABLES FROM THE CALLING PROGRAM:
C      PERIOD, NPRT, PW, N, J, SAMPLE.
C      IT RETURNS THE FOLLOWING VARIABLES TO THE CALLING PROGRAM:
C      FREQ.

C      'FM' IS COMPILED ON THE ECLIPSE USING THE COMMAND:
C      FORTRAN FM
C      THIS SUBROUTINE IS STORED IN FILE 'FM.FR' IN THE DIRECTORY DP4:REED.

C      THE FOLLOWING VARIABLES ARE USED IN 'FM':
C      J - SPECIFIES IF THE WAVEFORM IS FREQUENCY MODULATED OR NOT
C      BAND - BANDWIDTH OF THE FM
C      TIME - VALUE OF TAU (DELAY)
C      FREQ - ARRAY CONTAINING THE FM OF EACH SAMPLE
C      PERIOD- ARRAY CONTAINING THE # OF SAMPLES OF EACH PRT
C      PW - PULSE WIDTH
C      SAMPLE- # OF SAMPLES OF WAVEFORM PLUS THE ZERO-FILL SAMPLES NEEDED
C              TO PERFORM LINEAR CONVOLUTION
C      NPRT - # OF REPETITIONS OF PULSE MADE IN WAVEFORM
C      N - # OF SAMPLES MADE OF PULSE AFTER WEIGHTING

C      SUBROUTINE FM(FREQ, PERIOD, NPRT, PW, N, J, SAMPLE)

C      INTEGER PERIOD(10), SUM, SAMPLE, COUNT, REMAIN
C      REAL FREQ(1024)

C      IF (J EQ. 1) GO TO 40
C      TYPE 'WAVEFORM IS FREQUENCY MODULATED'
C      ACCEPT 'ENTER BANDWIDTH TO BE SWEEPED ', BAND
C      ACCEPT 'ENTER 0 IF LFM, 1 IF VFM ', I
C      IF (I EQ. 1) GO TO 15
C      WRITE(10, 300) BAND
C      SUM=0
C      DO 10 I=1, NPRT
C      COUNT=SUM+1
C      REMAIN=COUNT+N-1
C      DO 5 J=COUNT, REMAIN
C      TIME=(PW/N)*(J-SUM)
C      FREQ(J)=BAND*TIME*TIME*3.1416/PW
C      CONTINUE
C      SUM=SUM+PERIOD(I)

```

```

10    CONTINUE
      GO TO 50
15    WRITE(10,305) BAND
      SUM=0
      DO 30 I=1,NPRT
        COUNT=SUM+1
        REMAIN=COUNT+N-1
        DO 20 J=COUNT,REMAIN/2
          TIME=PW/N*(N-J*2-SUM)
          FREQ(J)=-0.5*BAND*TIME*TIME*3.1416/PW
20    CONTINUE
        DO 25 J=REMAIN/2+1,REMAIN
          TIME=PW/N*(J*2-N-SUM)
          FREQ(J)=0.5*BAND*TIME*TIME*3.1416/PW
25    CONTINUE
      SUM=SUM+PERIOD(I)
30    CONTINUE
      GO TO 50
40    TYPE 'WAVEFORM IS NOT FREQUENCY MODULATED'
      DO 45 I=1,SAMPLE
        FREQ(I)=0.0
45    CONTINUE
50    RETURN
300   FORMAT('LFM USED, BANDWIDTH TO BE SWEPT=',F12.3)
305   FORMAT('VFM USED, BANDWIDTH TO BE SWEPT=',F12.3)
      END

```

```

C*****
C*      SUBROUTINE GPULS      *
C*      WRITTEN BY: CAPT JOHN REED      *
C*      AFIT BOX 4027      *
C*****

C      GPULS IS A FORTRAN V SUBROUTINE WHICH DETERMINES THE AMPLITUDE
C      OF EACH SAMPLE OF THE RADAR WAVEFORM IF THE PULSE(S) IS GAUSSIAN SHAPED
C      (EXCEPT FOR PHASE-SHIFT CODED SIGNALS).
C
C      THE GPULS SUBROUTINE MAY BE CALLED BY THE FOLLOWING PROGRAMS:
C      MAIN, GMAIN, IMAIN
C      'GPULS' RECEIVES THE FOLLOWING VARIABLES FROM THE CALLING PROGRAM:
C      PERIOD, PW, NPRT, N.
C      IT RETURNS THE FOLLOWING VARIABLES TO THE CALLING PROGRAM:
C      Z, GFACTOR.
C
C      GPULS' IS COMPILED ON THE ECLIPSE USING THE COMMAND:
C      FORTRAN GPULS
C
C      THIS SUBROUTINE IS STORED IN FILE 'GPULS.FR' IN DIRECTORY DP4 REED
C
C      THE FOLLOWING VARIABLES ARE USED IN THIS SUBROUTINE:
C      MEAN - 1/2 LENGTH OF PW
C      STDEV - 1/3 OF MEAN; USE 3 STANDARD DEVIATIONS FROM MEAN
C      VAR - VARIANCE; STDEV*STDEV
C      GAUSS - SCALING VALUE OF GAUSSIAN PULSE
C      GFACTOR-SCALING VALUE FOR THE MAGNITUDE OF THE AMBIGUTY FCN
C      NPRT - # OF PRTS IN WAVEFORM
C      PW - PULSE WIDTH
C      N - # OF SAMPLES MADE OF PULSE AFTER WEIGHTING
C      Z - ARRAY CONTAINING THE AMPLITUDE OF EACH SAMPLE OF WAVEFORM
C      PERIOD- ARRAY CONTAINING # OF SAMPLES OF EACH PRT AFTER WEIGHTING

      SUBROUTINE GPULS(PERIOD, Z, PW, GFACTOR, NPRT, N)

      COMPLEX Z(1024)
      INTEGER SUM, COUNT, REMAIN, PERIOD(10)
      REAL MEAN

      TYPE 'PULSE(S) IS GAUSSIAN SHAPED'
      MEAN=0.5*PW
      STDEV=MEAN/3.0
      VAR=STDEV*STDEV
      GAUSS=1.0/SQRT(2.0*VAR*3.1416)
      GFACTOR=0.5923581*PW*PW
      WRITE(10,300) MEAN, STDEV, VAR, GFACTOR
      SUM=0
      DO 15 I=1, NPRT
      COUNT=SUM+1
      REMAIN=COUNT*N-1
      DO 5 J=COUNT, REMAIN
      TIME=(PW/N)*(J-SUM)
      POWER=TIME-MEAN

```

```

C*****
C*      SUBROUTINE RECPL      *
C*      WRITTEN BY: CAPT JOHN REED      *
C*      15 APRIL 1982      *
C*****

C      RECPL IS A FORTRAN V SUBROUTINE WHICH DETERMINES THE AMPLITUDE
C      OF EACH SAMPLE OF THE RADAR WAVEFORM IF THE PULSE(S) IS RECTANGULAR
C      SHAPED (EXCEPT FOR PHASE-SHIFT CODED SIGNALS).
C
C      THE RECPL SUBROUTINE MAY BE CALLED BY THE FOLLOWING PROGRAMS:
C      MAIN, GMAIN, IMAIN.
C      'RECPL' RECEIVES THE FOLLOWING VARIABLES FROM THE CALLING PROGRAM:
C      PERIOD, NPRT, N
C      IT RETURNS THE FOLLOWING VARIABLES TO THE CALLING PROGRAM:
C      Z, GFACTOR.
C
C      'RECPL' IS COMPILED ON THE ECLIPSE USING THE COMMAND:
C      FORTRAN RECPL
C
C      THIS SUBROUTINE IS STORED IN FILE 'RECPL.FR' IN DIRECTORY DP4.REED.
C
C      THE FOLLOWING VARIABLES ARE USED IN THIS SUBROUTINE:
C      GFACTOR-SCALING VALUE FOR THE MAGNITUDE OF THE AMBIGUITY FCN
C      NPRT - # OF PRTS IN THE WAVEFORM
C      Z - ARRAY CONTAINING THE AMPLITUDE OF EACH SAMPLE OF THE WAVEFORM
C      PERIOD- ARRAY CONTAINING THE # OF SAMPLES OF EACH PRT AFTER WEIGHTING
C      N - # OF SAMPLES OF PULSE AFTER WEIGHTING

      SUBROUTINE RECPL(PERIOD, Z, GFACTOR, NPRT, N)

      COMPLEX Z(1024)
      INTEGER SUM, COUNT, REMAIN, PERIOD(10)

      SUM=0
      GFACTOR=1.0
      TYPE 'PULSE(S) IS RECTANGULAR SHAPED'
      DO 15 I=1, NPRT
        COUNT=SUM+1
        REMAIN=COUNT+N-1
        DO 5 J=COUNT, REMAIN
          Z(J)=(1.0, 0.0)
          CONTINUE
        DO 10 J=REMAIN+1, SUM+PERIOD(I)
          Z(J)=(0.0, 0.0)
          CONTINUE
        SUM=SUM+PERIOD(I)
      15 CONTINUE
      RETURN
      END

```

```

      Z(J)=CMPLX(GAUSS*EXP(-POWER*POWER/(2.0*VAR)),0.0)
5      CONTINUE
      DO 10 J=REMAIN+1,SUM+PERIOD(I)
      Z(J)=(0.0,0.0)
10     CONTINUE
      SUM=SUM+PERIOD(I)
15     CONTINUE
      RETURN
300    FORMAT('M=',F12.10,3X,'SD=',F12.10,3X,'VAR=',F12.10,3X,'GFAC=',F12.10)
      END

```

```

C*****
C*      SUBROUTINE SETUP      *
C*      WRITTEN BY: CAPT JOHN REED      *
C*      15APRIL 1982      *
C*****

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```

C      'SETUP' IS A FORTRAN V SUBROUTINE WHICH SETS UP ARRAYS WITH THE
C      TIME-REVERSED CONJUGATE OF THE COMPLEX ENVELOPE (XVALS) AND THE DELAYED,
C      DOPPLER-SHIFTED VALUES OF THE COMPLEX ENVELOPE(YVALS). THIS WILL
C      THEN ALLOW US TO PERFORM THE LINEAR CONVOLUTION LATER ON.

```

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C      THIS SUBROUTINE IS CALLED BY THE 'MAIN' PROGRAM AND THE 'PHOTO'
C      AND 'INTEG' SUBROUTINES.
C      THIS SUBROUTINE RECEIVES THE FOLLOWING VARIABLES FROM THE CALLING ROUTINE:
C      Z, FREQ, VALUE, TOTAL, SAMPLE, M.
C      THIS SUBROUTINE RETURNS THE FOLLOWING VARIABLES TO THE CALLING ROUTINE:
C      XVALS, YVALS.

```

```

C      'SETUP' IS COMPILED ON THE ECLIPSE USING THE COMMAND:
C      FORTRAN SETUP.

```

```

C      'SETUP' IS STORED IN FILE 'SETUP.FR' IN DIRECTORY DP4 REED.

```

```

C      THE FOLLOWING VARIABLES ARE USED IN THIS SUBROUTINE:
C      M -      # OF NON-ZERO-FILL SAMPLES OF WAVEFORM AFTER WEIGHTING
C      SAMPLE- # OF SAMPLES OF WAVEFORM WITH ALL ZERO-FILL SAMPLES
C      YVALS - ARRAY CONTAINING THE DELAYED, DOPPLER-SHIFTED SAMPLES OF THE
C      COMPLEX ENVELOPE OF THE WAVEFORM
C      XVALS - ARRAY CONTAINING THE TIME-REVERSED, CONJUGATE SAMPLES OF THE
C      COMPLEX ENVELOPE OF THE WAVEFORM
C      FREQ - ARRAY CONTAINING THE FM OF EACH SAMPLE
C      TIME - VALUE OF TAU (DELAY)
C      TOTAL - TOTAL TIME DURATION OF THE WAVEFORM
C      VALUE - IMAGINARY VALUE = DOPPLER FREQUENCY(W)
C      Z -      ARRAY CONTAINING THE AMPLITUDE OF EACH SAMPLE OF WAVEFORM

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      SUBROUTINE SETUP(Z, FREQ, XVALS, YVALS, VALUE, TOTAL, SAMPLE, M)

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```

      COMPLEX XVALS(1024), YVALS(1024), Z(1024), VALUE, AMT1, AMT2
      INTEGER SAMPLE
      REAL FREQ(1024)

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```

      DO 5 I=M+1, SAMPLE
      YVALS(I)=(0.0,0.0)
5      CONTINUE
      DO 10 I=1, SAMPLE-M
      XVALS(I)=(0.0,0.0)
10     CONTINUE
      DO 15 I=1, M
      AMT2=CMPLX(0.0, FREQ(I))
      TIME=TOTAL/M*I
      YVALS(I)=Z(I)*CEXP(VALUE*TIME+AMT2)
15     CONTINUE
      DO 20 I=SAMPLE-M+1, SAMPLE

```

20      AMT1=Z(I-SAMPLE+M)  
          AMT2=CMPLX(0.0,-FREQ(I-SAMPLE+M))  
          XVALS(I)=AMT1\*CEXP(AMT2)  
          CONTINUE  
          RETURN  
          END

```

C*****
C*      SUBROUTINE OUT      *
C*      WRITTEN BY: CAPT JOHN REED      *
C*      15 APRIL 1982      *
C*****

C      'OUT' IS A FORTRAN V SUBROUTINE WHICH DISPLAYS THE MAGNITUDE
C      OF THE AMBIGUITY FUNCTION AT VARIOUS DELAY TIMES (VALUES OF TAU).
C
C      'OUT' IS CALLED BY THE 'MAIN' PROGRAM.
C
C      'OUT' RECEIVES THE FOLLOWING VARIABLES FROM 'MAIN':
C      MAG, NUM, PW, TOTAL, M.
C      'OUT' DOES NOT RETURN ANY VARIABLE TO 'MAIN'.
C
C      'OUT' IS COMPILED ON THE ECLIPSE USING THE COMMAND:
C      FORTRAN OUT
C
C      'OUT' IS STORED IN FILE 'OUT.FR' IN DIRECTORY DP4:REED.
C
C      THE FOLLOWING VARIABLES ARE USED IN 'OUT':
C      PW - PULSE WIDTH
C      NUM - # OF SAMPLES MADE OF WAVEFORM
C      M - # OF SAMPLES MADE OF WAVEFORM AFTER WEIGHTING
C      TOTAL - TOTAL TIME DURATION OF WAVEFORM
C      MAG - ARRAY CONTAINING THE MAGNITUDE OF EACH SAMPLE OF THE
C            AMBIGUITY FCN CUT

      SUBROUTINE OUT(MAG, NUM, PW, TOTAL, M)

      REAL MAG(1024)

      WRITE(10,300) PW, NUM
      WRITE(10,305)
      DO 5 I=1, M
      TIME=(TOTAL/M*(I-1))-TOTAL
      WRITE(10,310) TIME, MAG(I)
      CONTINUE
      DO 10 I=M+1, M*2
      TIME=TOTAL/M*(I-M-1)
      WRITE(10,310) TIME, MAG(I)
      CONTINUE
      RETURN
300  FORMAT('PW=', F12.10, 5X, '# OF SAMPLES MADE OF WAVEFORM=', I5)
305  FORMAT('TIME(DELAY)', 5X, 'MAGNITUDE OF AMBIGUITY FUNCTION')
310  FORMAT(F12.10, 7X, F10.6)
      END

```



```

C*****
C*      PROGRAM GMAIN      *
C*      WRITTEN BY: CAPT JOHN REED      *
C*      15 APRIL 1982      *
C*****

C      'GMAIN' IS A FORTRAN V PROGRAM WHICH CALCULATES THE MAGNITUDE OF THE
C      AMBIGUITY FUNCTION FOR A SPECIFIED RADAR WAVEFORM. THE MAGNITUDE IS
C      CALCULATED FOR 64 DIFFERENT DOPPLER FREQUENCIES, BEGINNING AND ENDING
C      WITH FREQUENCIES THE USER SPECIFIES AND AT EQUAL INTERVALS IN BETWEEN.
C      THESE MAGNITUDES ARE QUANTIZED INTO VALUES RANGING FROM 0 TO 200.
C      THEN, UP TO 64 SAMPLES OF EACH OF THESE QUANTIZED MAGNITUDES ARE
C      WRITTEN INTO A FILE 'DATA'. THESE DATA ARE THEN USED BY ANOTHER
C      PROGRAM 'PLTTRNS' TO PROVIDE A THREE-DIMENSIONAL PLOT OF THE MAGNITUDE
C      OF THE AMBIGUITY FUNCTION.

C      THE FOLLOWING ARE USER SELECTED PARAMETERS WHICH THE PROGRAM (OR
C      THE SUBROUTINES WHICH IT CALLS) WILL REQUEST:
C      1 PULSE WIDTH.
C      2 WHETHER OR NOT THE PULSE IS REPEATED.
C          A. WHETHER PULSE REPETITION IS STAGGERED OR THE SAME.
C          B. PULSE REPETITION TIME(S) (PRT).
C          C. NUMBER OF REPETITIONS.
C      3 NUMBER OF SAMPLES TO BE MADE OF WAVEFORM.
C      4 WHETHER OR NOT THE PULSE(S) IS PHASE-SHIFT CODED.
C          A. NUMBER OF ELEMENTS IN THE PULSE CODE.
C          B. PHASE OF EACH ELEMENT.
C      5 WHETHER OR NOT THE PULSE(S) IS FREQUENCY MODULATED.
C          A. BANDWIDTH TO BE SWEEPED BY THE FM.
C          B. IF LINEAR FM OR VFM.
C      6 WHETHER THE PULSE(S) IS RECTANGULAR OR GAUSSIAN SHAPED.
C      7 STARTING VALUE OF DELAY (TAU) FOR THE SAMPLES OF THE MAGNITUDE
C      TO BE WRITTEN INTO THE 'DATA' FILE.
C      8 WHETHER OR NOT TO SKIP SAMPLES, AND IF SO, HOW MANY.
C      9 STARTING & FINAL VALUES OF THE DOPPLER FREQUENCIES FOR WHICH
C      MAGNITUDES ARE CALCULATED.

C      'GMAIN' MAY MAKE CALLS TO THE FOLLOWING SUBROUTINES:
C      INPUT, PHASE, FM, GPULS, RECPL, PHOTO (WHICH CALLS THE FAST FOURIER
C      TRANSFORM SUBROUTINE (EDFT LB) FROM THE LIBRARY).

C      'GMAIN' IS COMPILED AND LOADED ON THE ECLIPSE USING THE FOLLOWING
C      COMMANDS:
C      FORTRAN GMAIN
C      RLDR/N GMAIN INPUT PHASE FM GPULS RECPL PHOTO EDFT.LB @FLIB@

C      THIS PROGRAM IS STORED IN FILE 'GMAIN.FR' IN DIRECTORY DP4:REED.

C      THE FOLLOWING VARIABLES ARE USED IN 'GMAIN':
C      PW - PULSE WIDTH
C      SAME - PRT IF PULSES ARE REPEATED AT SAME INTERVAL
C      NPRT - # OF REPETITIONS OF PULSE TO BE MADE
C      PRT - ARRAY CONTAINING THE PRT FOR EACH PULSE REPETITION
C      TOTAL - TOTAL TIME DURATION OF WAVEFORM

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C      NUM - # OF SAMPLES MADE OF WAVEFORM
C      SAMPLE- # OF SAMPLES OF WAVEFORM PLUS ZERO-FILLING SAMPLES NEEDED
C              TO PERFORM LINEAR CONVOLUTION (=2*NUM)
C      PERIOD- ARRAY CONTAINING # OF SAMPLES MADE OF EACH PRT
C      N - # OF SAMPLES MADE OF PULSE AFTER WEIGHTING
C      M - # OF NON-ZERO-FILL SAMPLES MADE OF WAVEFORM AFTER WEIGHTING
C      Z - ARRAY CONTAINING THE AMPLITUDE OF EACH SAMPLE
C      GFACTOR-SCALING VALUE OF MAGNITUDE
C      FREQ - ARRAY CONTAINING THE FM FOR EACH SAMPLE
C      SCALE - SCALING FACTOR OF MAGNITUDE

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COMPLEX Z(1024)
INTEGER SAMPLE,PERIOD(10)
REAL PRT(10),FREQ(1024)

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```

TYPE '          WELCOME TO GMAIN'
TYPE 'THIS PROGRAM IS DESIGNED TO COMPUTE THE MAGNITUDE OF 64'
TYPE 'DIFFERENT DOPPLER CUTS OF THE AMBIGUTY FUNCTION.'
TYPE 'QUANTIZE THE MAGNITUDES, AND OUTPUT UP TO 64 SAMPLES OF'
TYPE 'EACH CUT TO FILE DATA. YOU WILL BE REQUIRED TO ENTER'
TYPE 'PARAMETERS AT VARIOUS TIMES DURING THE PROGRAM. WHEN THE'
TYPE 'PROGRAM REQUESTS AN ENTRY, SIMPLY TYPE YOUR REPLY AND,'
TYPE 'IF CORRECT, HIT CR KEY. USE THE <DEL> KEY TO ERASE CHARACTERS'
TYPE 'AND THE <C> TO ERASE THE ENTIRE ENTRY. YOU MAY STOP THE'
TYPE 'PROGRAM AT ANY TIME BY ENTERING <^A>. YOU MAY STOP THE'
TYPE 'SCREEN OUTPUT USING <^S> AND RESUME USING <^Q>'
PAUSE 'ENTER ANY CHARACTER TO BEGIN'
CALL INPUT(PERIOD,PRT,PW,NPRT,TOTAL,SAMPLE,NUM,M,N)
ACCEPT 'ENTER 0 IF PULSE(S) IS PHASE-SHIFT CODED, 1 IF NOT ',J
IF (J EQ 1) GO TO 5
CALL PHASE(FREQ,PRT,PERIOD,Z,M,N,NPRT,GFACTOR,PW,SAMPLE,TOTAL)
GO TO 20
5 TYPE 'PULSE(S) IS NOT PHASE-SHIFT CODED'
ACCEPT 'ENTER 0 IF PULSE(S) IS FREQUENCY MODULATED, 1 IF NOT ',J
CALL FM(FREQ,PERIOD,NPRT,PW,N,J,SAMPLE)
10 ACCEPT 'ENTER 0 FOR RECTANGULAR-SHAPED PULSE(S), 1 FOR GAUSSIAN ',J
IF (J EQ 0) GO TO 15
CALL GPULS(PERIOD,Z,PW,GFACTOR,NPRT,N)
GO TO 20
15 CALL RECPL(PERIOD,Z,GFACTOR,NPRT,N)
20 SCALE=GFACTOR/(N*NPRT)
CALL PHOTO(Z,FREQ,TOTAL,M,SAMPLE,SCALE)
TYPE 'DATA HAVE BEEN TRANSFERRED TO FILE 'DATA''
TYPE 'AFTER THIS PROGRAM ENDS, ENTER THE FOLLOWING COMMAND ON'
TYPE 'THE TEKTRONIX SCOPE: PLTTRNS DATA/I (ARRAY SIZE)/N'
END

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```

C*****
C*      SUBROUTINE PHOTO      *
C*      WRITTEN BY: CAPT JOHN REED      *
C*      15 APRIL 1982      *
C*****

C      'PHOTO' IS A FORTRAN V SUBROUTINE WHICH TAKES A DESIRED RADAR
C      WAVEFORM, PERFORMS THE CALCULATIONS TO PRODUCE THE AMBIGUITY FCN
C      MAGNITUDES AT VARIOUS CUTS OF DOPPLER FREQUENCIES, QUANTIZES THE
C      MAGNITUDES TO VALUES BETWEEN 0 AND 200, SELECTS THE SAMPLES OF THE
C      MAGNITUDES TO BE OBSERVED, AND WRITES THESE DATA INTO FILE 'DATA'.
C
C      'PHOTO' IS CALLED BY THE PROGRAM 'GMAIN'.
C      'PHOTO' MAKES CALLS TO THE 'SETUP' SUBROUTINE AND TO
C      THE FFT LIBRARY SUBROUTINE EDFT.LB (DFT5).
C      'PHOTO' RECEIVES THE FOLLOWING VARIABLES WHEN CALLED BY 'GMAIN':
C      Z, FREQ, TOTAL, M, SAMPLE, SCALE.
C      'PHOTO' DOES NOT RETURN ANY VARIABLES TO 'GMAIN'.
C
C      'PHOTO' IS COMPILED ON THE ECLIPSE USING THE COMMAND:
C      FORTRAN PHOTO
C      'PHOTO' IS STORED IN FILE 'PHOTO FR' IN DIRECTORY DP4 REED.
C
C      THE FOLLOWING VARIABLES ARE USED IN 'PHOTO':
C      INTERVAL-TIME (TAU) BETWEEN EACH SAMPLE OF WAVEFORM
C      M -      # OF NON-ZERO-FILL SAMPLES MADE OF WAVEFORM AFTER WEIGHTING
C      JL -      # OF SAMPLES OF MAGNITUDE OF AMB FCN FOR EACH DOPPLER
C      TOTAL - TOTAL TIME DURATION OF WAVEFORM
C      A -      STARTING VALUE OF TIME FOR MAGNITUDE OF AMB FCN FOR EACH CUT
C      N -      HOW TO SAMPLE AVAILABLE SAMPLES OF MAGNITUDE
C      START - STARTING SAMPLE TO BE OBSERVED
C      STOP - FINAL SAMPLE TO BE OBSERVED
C      WS -      STARTING DOPPLER FREQUENCY TO BE CONSIDERED
C      WF -      FINAL DOPPLER FREQUENCY TO BE CONSIDERED
C      WIN -      INTERVAL BETWEEN DOPPLER FREQUENCIES TO BE CONSIDERED
C      W -      ARRAY CONTAINING ALL DOPPLER FREQUENCIES TO BE CONSIDERED
C      IER -      ERROR LABEL FOR A FILE OPERATION (OKAY IF = 1)
C      Z -      ARRAY CONTAINING THE AMPLITUDE FOR ALL SAMPLES OF WAVEFORM
C      FREQ -      ARRAY CONTAINING THE FMOF ALL WAVEFORM SAMPLES
C      XVALS -      ARRAY CONTAINING THE TIME-REVERSED, CONJUGATE SAMPLES OF
C      THE COMPLEX ENVELOPE OF THE WAVEFORM; ALSO CONTAINS THE
C      AMBIGUITY FCN VALUES LATER IN THE ROUTINE
C      YVALS -      ARRAY CONTAINING THE DOPPLER AND RANGE-DELAYED SAMPLES OF
C      THE COMPLEX ENVELOPE OF WAVEFORM
C      VALUE -      IMAGINARY VALUE = W
C      SAMPLE-      # OF SAMPLES OF WAVEFORM PLUS ZERO-FILLING SAMPLES NEEDED
C      TO PERFORM LINEAR CONVOLUTION
C      MAG -      ARRAY CONTAINING THE SAMPLES OF THE MAGNITUDE OF THE
C      AMBIGUITY FCN FOR A DOPPLER CUT
C      G -      AN UNGUANTIZED VALUE OF MAGNITUDE
C      K -      A QUANTIZED VALUE OF MAGNITUDE (BETWEEN 0 AND 200)

SUBROUTINE PHOTO(Z, FREQ, TOTAL, M, SAMPLE, SCALE)
COMPLEX XVALS(1024), YVALS(1024), Z(1024), VALUE, STORE(64)

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INTEGER DIFF, A, START, STOP, GRAPH(64, 64), SAMPLE
REAL INTERVAL, W(64), FREQ(1024), MAG(1024)

INTERVAL=TOTAL/M
WRITE(10,300) -TOTAL, TOTAL, INTERVAL
ACCEPT 'CENTER VALUE OF TAU IS A*(TOTAL/M), ENTER A ', A
ACCEPT 'TO GRAPH EACH SAMPLE, ENTER 1; EVERY OTHER SAMPLE, 2, ETC. ', N
WRITE(10,305) A*INTERVAL
NUM=SAMPLE/2
DIFF=NUM-M
IF ((A+NUM-32*N) GT. 0) GO TO 1
START=A-(32*N)+NUM
STOP=A+(32*N)+NUM
GO TO 2
1  START=A-(32*N)+M
   STOP=A+(32*N)+M
2  IF (START LT. 0) START=0
   IF (STOP GT. SAMPLE) STOP=SAMPLE
   WRITE(10,310) (START-M)*INTERVAL
   WRITE(10,315) (STOP-NUM+DIFF)*INTERVAL
   JL=(STOP-START)/N
   IF (JL NE. 64) TYPE 'ERROR IN SAMPLING SELECTION'
   ACCEPT 'ENTER STARTING VALUE OF W ', WS
   ACCEPT 'ENTER FINAL VALUE OF W ', WF
   WIN=(WF-WS)/64.0
   WRITE(10,320) (WF+WS)/2.0
   DO 5 I=1, 64
     W(I)=WS+WIN*I
5  CONTINUE
   CALL DFILW("DATA", IER)
   TYPE 'FOR DFILW, IER=', IER
   CALL CFILW("DATA", 2, IER)
   TYPE 'FOR CFILW, IER=', IER
   CALL OPEN(6, "DATA", 2, IER)
   TYPE 'FOR OPEN, IER=', IER
   DO 25 I=1, 64
     WRITE(10,325) W(I)
     VALUE=CMPLX(0.0, W(I))
     CALL SETUP(2, FREQ, XVALS, YVALS, VALUE, TOTAL, SAMPLE, M)
     CALL DFT5(XVALS, SAMPLE, 0)
     CALL DFT5(YVALS, SAMPLE, 0)
     DO 10 J=1, SAMPLE
       XVALS(J)=CONJG(XVALS(J))
       YVALS(J)=CONJG(YVALS(J))
       XVALS(J)=XVALS(J)*YVALS(J)/SAMPLE
10  CONTINUE
     CALL DFT5(XVALS, SAMPLE, 1)
     DO 15 J=1, SAMPLE
       MAG(J)=CABS(XVALS(J))*SCALE
15  CONTINUE
     WRITE(10,330) W(I)
     DO 20 J=START+N, STOP, N
       G=MAG(J)
     K=0

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      DO 21 L=1,200
      R=0.005*L
      IF (G GE R) K=L
21      CONTINUE
      GRAPH((J-START)/N, I)=K
20      CONTINUE
25      CONTINUE
      DO 35 I=1,64
      DO 30 J=1,JL
      STORE(J)=GRAPH(J, I)
30      CONTINUE
      CALL WRBLK(6, I, STORE, 1, IER)
35      CONTINUE
      CALL CLOSE(6, IER)
      TYPE 'FOR CLOSE, IER=', IER
      WRITE(10, 310) (START-M)*INTERVAL
      WRITE(10, 315) (STOP-NUM+DIFF)*INTERVAL
      TYPE 'ARRAY SIZE=', JL
      RETURN
300      FORMAT('TAU GOES FROM ', F12.10, ' TO ', F12.10, ' IN INTERVALS OF ', F12.10)
305      FORMAT('TAU IS CENTERED AT ', F12.10)
310      FORMAT('TAU STARTS AT ', F12.9)
315      FORMAT('TAU STOPS AT ', F12.9)
320      FORMAT('W IS CENTERED AT ', F12.2)
325      FORMAT('W NOW BEING EVALUATED = ', F12.2)
330      FORMAT('FOR W = ', F12.2, 3X, 'MAGNITUDES HAVE BEEN CALCULATED')
      END

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C*****
C*      PROGRAM IMAIN      *
C*      WRITTEN BY: CAPT JOHN REED      *
C*      15 APRIL 1982      *
C*****

C      'IMAIN' IS A FORTRAN V PROGRAM WHICH CALCULATES THE SIGNAL-TO-CLUTTER
C      RATIO FOR A SPECIFIED RADAR WAVEFORM, CLUTTER DISTRIBUTION, AND
C      DESIRED TARGET. APPROXIMATIONS ARE MADE IN CALCULATING THE VOLUME OF
C      THE PRODUCT OF THE AMBIGUITY FCN MAGNITUDE AND THE CLUTTER PROBABILITY
C      DENSITY FCN. IN THIS PROGRAM THE STANDARD PROBABILITY DENSITY FCNS FOR
C      CLUTTER ARE UNIFORM AND GAUSSIAN. THE PROGRAM ALLOWS THE USER TO
C      INPUT ANY OTHER NON-STANDARD DENSITY FOR CLUTTER.
C
C      THE FOLLOWING ARE USER SELECTED PARAMETERS WHICH THE PROGRAM
C      (OR THE SUBROUTINES IT CALLS) WILL REQUEST:
C      1. PULSE WIDTH.
C      2. WHETHER OR NOT THE PULSE IS REPEATED.
C          A. WHETHER PULSE REPETITION IS STAGGERED OR THE SAME.
C          B. PULSE REPETITION TIME(S) (PRT).
C          C. NUMBER OF REPETITIONS.
C      4. WHETHER OR NOT THE PULSE(S) IS PHASE-SHIFT CODED.
C          A. NUMBER OF ELEMENTS IN THE PULSE CODE.
C          B. PHASE OF EACH ELEMENT.
C      5. WHETHER OR NOT THE PULSE(S) IS FREQUENCY MODULATED.
C          A. BANDWIDTH TO BE SWEEPED BY THE FM.
C          B. IF LINEAR FM OR VFM.
C      6. WHETHER PULSE(S) IS RECTANGULAR OR GAUSSIAN SHAPED.
C      7. STARTING DOPPLER FREQUENCY FOR CLUTTER & AMBIGUITY FCN.
C      8. FINAL DOPPLER FREQUENCY FOR CLUTTER & AMBIGUITY FCN.
C      9. INTERVAL BETWEEN DOPPLER FREQUENCIES.
C      10. PROBABILITY DENSITY FCN FOR THE CLUTTER.
C      11. BOUNDS ON DELAY (TAU) OF CLUTTER.
C      12. IF CLUTTER IS NOT A STANDARD PROB DENSITY FCN, THEN
C          A. # OF AREAS OF CLUTTER.
C          B. BOUNDS OF EACH CLUTTER AREA IN DELAY(TAU) & DOPPLER.
C      13. IF CLUTTER IS A STANDARD PROB DENSITY FCN, THEN
C          A. NECESSARY STATISTICS OF DOPPLER TO DEFINE DENSITY FCN.
C          B. NECESSARY STATISTICS OF DELAY(TAU) TO DEFINE DENSITY FCN.
C
C      'IMAIN' MAY MAKE CALLS TO THE FOLLOWING SUBROUTINES:
C      INPUT, PHASE, FM, GPULS, RECPL, INTEG (WHICH CALLS SEVERAL SUBROUTINES).
C
C      'IMAIN' IS COMPILED AND LOADED ON THE ECLIPSE USING THE FOLLOWING
C      COMMANDS.
C          FORTRAN IMAIN
C          RLDR/N IMAIN INPUT PHASE FM GPULS RECPL INTEG IDC CPDID SETUP PROB
C          EDFT LB @FLIB@
C
C      THIS PROGRAM IS STORED IN FILE 'IMAIN.FR' IN DIRECTORY DP4 REED.
C
C      THE FOLLOWING VARIABLES ARE USED IN 'IMAIN'
C      PERIOD - ARRAY CONTAINING # OF SAMPLES MADE OF EACH PRT
C      PRT - ARRAY CONTAINING THE PRT FOR EACH REPETITION

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C      PW - PULSE WIDTH
C      NPRT - # OF REPETITIONS OF PULSE TO BE MADE
C      TOTAL - TOTAL TIME DURATION OF WAVEFORM
C      NUM - # OF SAMPLES TO BE MADE OF WAVEFORM
C      SAMPLE- # OF SAMPLES MADE OF WAVEFORM PLUS ZERO-FILL SAMPLES NEEDED
C              TO PERFORM LINEAR CONVOLUTION (=2*NUM)
C      M - # OF SAMPLES MADE OF WAVEFORM AFTER WEIGHTING
C      N - # OF SAMPLES MADE OF PULSE AFTER WEIGHTING
C      FREQ - ARRAY CONTAINING THE FM OF EACH SAMPLE OF WAVEFORM
C      Z - ARRAY CONTAINING THE AMPLITUDE OF EACH SAMPLE OF WAVEFORM
C      GFACTOR-SCALING VALUE OF MAGNITUDE
C      SCALE - SCALING FACTOR OF MAGNITUDE
C      VALUE - IMAGINARY VALUE = W
C      TRCS - RADAR CROSS SECTION (RCS) OF DESIRED TARGET
C      CRCS - AVERAGE RCS OF THE CLUTTER
C      AVGVOL- AVERAGE VOLUME OF THE PRODUCT OF THE AMB FCN MAG AND
C              CLUTTER PROB DENSITY FCN
C      NS - TOTAL # OF SCATTERERS
C      SCR - SIGNAL-TO-CLUTTER RATIO

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COMPLEX Z(1024)
INTEGER SAMPLE,PERIOD(10)
REAL PRT(10),FREQ(1024)

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TYPE ' WELCOME TO IMAIN'
TYPE 'THIS PROGRAM IS DESIGNED TO COMPUTE THE SIGNAL-TO-CLUTTER'
TYPE 'RATIO FOR A SPECIFIED RADAR WAVEFORM, CLUTTER DISTRIBUTION,'
TYPE 'AND TARGET. YOU WILL BE REQUIRED TO ENTER PARAMETERS AT'
TYPE 'VARIOUS TIMES DURING THE PROGRAM. WHEN THE PROGRAM REQUESTS'
TYPE 'AN ENTRY, SIMPLY TYPE YOUR REPLY, AND IF CORRECT, HIT THE '
TYPE 'CR KEY. USE THE <DEL> KEY TO DELETE CHARACTERS AND THE <\>'
TYPE 'TO DELETE THE ENTIRE ENTRY. USE <^A> TO STOP THE PROGRAM.'
TYPE 'USE <^S> TO STOP THE SCREEN DISPLAY, AND <^Q> TO RESUME.'
PAUSE 'ENTER ANY CHARACTER TO BEGIN'
CALL INPUT(PERIOD,PRT,PW,NPRT,TOTAL,SAMPLE,NUM,M,N)
ACCEPT 'ENTER 0 IF PULSE(S) IS PHASE-SHIFT CODED, 1 IF NOT ',J
IF (J EQ 1) GO TO 5
CALL PHASE(FREQ,PRT,PERIOD,Z,M,N,NPRT,GFACTOR,PW,SAMPLE,TOTAL)
GO TO 20
5 TYPE 'PULSE(S) IS NOT PHASE-SHIFT CODED'
ACCEPT 'ENTER 0 IF PULSE(S) IS FREQUENCY MODULATED, 1 IF NOT ',J
CALL FM(FREQ,PERIOD,NPRT,PW,N,J,SAMPLE)
10 ACCEPT 'ENTER 0 FOR RECTANGULAR-SHAPED PULSE(S), 1 FOR GAUSSIAN ',J
IF (J EQ 0) GO TO 15
CALL GPULS(PERIOD,Z,PW,GFACTOR,NPRT,N)
GO TO 20
15 CALL RECPL(PERIOD,Z,GFACTOR,NPRT,N)
20 SCALE=GFACTOR/(N*NPRT)
CALL INTEG(Z,FREQ,TOTAL,SAMPLE,M,AVGVOL,SCALE)
ACCEPT 'ENTER THE RADAR CROSS SECTION (RCS) OF DESIRED TARGET ',TRCS
ACCEPT 'ENTER THE AVERAGE RCS OF THE CLUTTER ',CRCS
ACCEPT 'ENTER THE TOTAL NUMBER OF SCATTERERS ',NS
SCR=TRCS/(NS*CRCS*AVGVOL)
WRITE(10,300) TRCS,CRCS

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WRITE(10,305) NS,AVGVOL  
WRITE(10,310) SCR  
300 FORMAT('TARGET RCS=',F10.6,5X,'AVG RCS OF CLUTTER=',F10.6)  
305 FORMAT('TOTAL # OF SCATTERERS=',I5,5X,'AVG VOL OF INTEGRAL=',F10.6)  
310 FORMAT('SIGNAL-TO-CLUTTER RATIO=',F10.6)  
END
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C*****
C*      SUBROUTINE INTEG      *
C*      WRITTEN BY: CAPT JOHN REED      *
C*      15 APRIL 1982      *
C*****

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C      'INTEG' IS A FORTRAN V SUBROUTINE WHICH TAKES A RADAR WAVEFORM,
C      TAKES THE CLUTTER DISTRIBUTION, AND CALCULATES THE VOLUME OF
C      THE PRODUCT OF THE MAGNITUDE OF THE AMBIGUTY FCN AND THE CLUTTER
C      PROBABILITY DENSITY FCN. IT CALCULATES THE MAGNITUDES OF THE AMB
C      FCN FOR THE CUTS OF DOPPLER FREQUENCIES SPECIFIED BY THE USER. THE
C      VOLUME IS APPROXIMATED BY SUMMING THE PRODUCTS OF INCREMENTAL AREAS OF
C      THE DOPPLER AND DELAY BOUNDS WITH THE CLUTTER DENSITY FOR THOSE AREAS.
C      THE ACTUAL ALGORITHM IS TO TAKE TWO ADJACENT SAMPLES IN TIME OF
C      THE MAGNITUDES OF TWO ADJACENT AMBIGUITY FUNCTION CUTS. THE VALUE
C      OF THE PROBABILITY DENSITY FUNCTION AT EACH OF THESE FOUR POINTS IS
C      DETERMINED AND MULTIPLIED WITH THE CORRESPONDING VALUES OF THE
C      AMBIGUITY FUNCTION MAGNITUDES. THE MAXIMUM VALUE, MINIMUM VALUE,
C      AND AVERAGE VALUE OF THESE FOUR PRODUCTS IS DETERMINED AND MULTIPLIED
C      BY THE INCREMENTAL AREA. THIS RESULTS IN AN INCREMENTAL VALUE OF
C      MAXIMUM VOLUME, MINIMUM VOLUME, AND AVERAGE VOLUME. THE SAMPLES OF
C      THE MAGNITUDE OF THE AMBIGUITY FUNCTION CUTS ARE SHIFTED BY ONE AND
C      THE PROCESS IS REPEATED UNTIL ALL SAMPLES HAVE BEEN CONSIDERED. THEN
C      THE AMBIGUTY FUNCTION CUTS ARE SHIFTED BY ONE AND THE PROCESS OF
C      SAMPLING THE MAGNITUDES IS REPEATED. AFTER ALL DOPPLER CUTS HAVE
C      BEEN CONSIDERED, THE SUM OF THE INCREMENTAL VOLUMES GIVES THE OVERALL
C      VOLUME OF THE PRODUCT.

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C      'INTEG' MAY MAKE CALLS TO THE FOLLOWING SUBROUTINES.
C      IDC, CPDID, SETUP, PROB, & THE FFT LIBRARY SUBROUTINE EDFT LB(DFTS).
C      'INTEG' RECEIVES THE FOLLOWING VARIABLES FROM THE PROGRAM 'IMAIN':
C      Z, FREQ, TOTAL, SAMPLE, M, SCALE.
C      'INTEG' RETURNS THE VARIABLE 'AVGVOL' TO 'IMAIN'.
C
C      'INTEG' IS COMPILED ON THE ECLIPSE USING THE COMMAND:
C      FORTRAN INTEG

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C      THIS SUBROUTINE IS STORED IN FILE 'INTEG.FR' IN DIRECTORY DP4 REED.
C
C      THE FOLLOWING VARIABLES ARE USED IN 'INTEG':
C      Z - ARRAY CONTAINING THE AMPLITUDE OF EACH SAMPLE OF THE CUT
C      FREQ - ARRAY CONTAINING THE FM OF EACH SAMPLE OF A CUT
C      TOTAL - TOTAL TIME DURATION OF THE WAVEFORM
C      SAMPLE- # OF SAMPLES MADE OF WAVEFORM PLUS THE ZERO-FILL
C      SAMPLES NEEDED TO PERFORM THE LINEAR CONVOLUTION
C      M - # OF SAMPLES OF WAVEFORM AFTER WEIGHTING
C      SCALE - SCALING FACTOR OF MAGNITUDE
C      WS - STARTING VALUE OF DOPPLER FREQUENCIES FOR CLUTTER & AMB FCN
C      WF - FINAL VALUE OF DOPPLER FREQS FOR CLUTTER & AMB FCN
C      WIN - INCREMENT OF DOPPLER FREQ BETWEEN WS & WF
C      NUMW - # OF DOPPLER FREQS TO BE CONSIDERED
C      AREA - INCREMENTAL AREA OF DOPPLER AND DELAY: (TIME (TAU) BETWEEN
C      SAMPLES OF AMB FCN MAG)*(INTERVAL BETWEEN DOPPLER FREQS)
C      MAXVOL- MAXIMUM VALUE OF THE VOLUME OF THE PRODUCT OF THE

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C          AMB FCN MAG & CLUTTER DENSITY
C      MINVOL -MINIMUM VALUE OF THE VOLUME
C      AVGVOL- AVERAGE VALUE OF THE VOLUME
C      IP -    USED TO IDENTIFY NON-STANDARD CLUTTER PROB DENSITY FCN
C      CTS -   LOWER BOUND OF TAU FOR CLUTTER
C      CTF -   UPPER BOUND OF TAU FOR CLUTTER
C      DCWF -  ARRAY CONTAINING FINAL VALUE OFDOPPLER FOR EACH CLUTTER AREA
C      DCWS -  ARRAY CONTAINING STARTING DOPPLER VALUE FOR EACH CLUTTER AREA
C      DCTS -  ARRAY CONTAINING STARTING TAU VALUE FOR EACH CLUTTER AREA
C      DCTF -  ARRAY CONTAINING FINAL TAU VALUE FOR EACH CLUTTER AREA
C      WMEAN - STATISTIC USED TO DESCRIBE STANDARD PROB DENSITY FCN
C      WSTDV - STATISTIC OF W USED TO DESCRIBE A STANDARD PROB DENSITY FCN
C      TMEAN - STATISTIC OF TAU USED TO DESCRIBE A STANDARD PROB DENSITY FCN
C      TSTDV - STATISTIC OF TAU USED TO DESCRIBE A STANDARD PROB DENSITY FCN
C      W -     DOPPLER FREQ OF CUT TO BE MADE
C      VALUE - IMAGINARY VALUE = W
C      XVALS - ARRAY CONTAINING THE TIME-REVERSED, CONJUGATE SAMPLES OF THE
C              COMPLEX ENVELOPE OF WAVEFORM, LATER CONTAINS SAMPLES
C              OF THE CUT OF THE AMB FCN
C      YVALS - ARRAY CONTAINING THE DOPPLER AND TIME-DELAYED SAMPLES OF THE
C              COMPLEX ENVELOPE OF WAVEFORM
C      MAG -   ARRAY CONTAINING THE MAGNITUDE SAMPLES OF A CUT OF AMB FCN
C      TIME -  VALUE OF TIME(TAU)
C      DENSITY-VALUE OF THE CLUTTER PROB DENSITY FCN FOR A SPECIFIED W & TAU
C      CSUM -  TOTAL VALUE OF THE PROB DENSITY FCN FOR NON-STANDARD
C              CLUTTER DISTRIBUTION
C      PROD -  ARRAY CONTAINING THE PRODUCT OF THE PRODUCT OF THE
C              AMB FCN MAG & CLUTTER DENSITY
C      MIN -   USED TO STORE MINIMUM VALUE OF THE FOUR SAMPLES
C      MAX -   USED TO STORE MAXIMUM VALUE OF THE FOUR SAMPLES
C      ICDY -  IDENTIFIES DELAY PROB DENSITY FCN FOR CLUTTER
C      ICDP -  IDENTIFIES DOPPLER PROB DENSITY FCN FOR CLUTTER

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SUBROUTINE INTEG(Z,FREQ,TOTAL,SAMPLE,M,AVGVOL,SCALE)

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COMPLEX XVALS(1024),YVALS(1024),Z(1024),VALUE
INTEGER SAMPLE
REAL MAX,MIN,MAXVOL,MINVOL,MAG(1024),PROD(1024,2),FREQ(1024)
      ,DCWS(10),DCWF(10),DCTS(10),DCTF(10)

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ACCEPT "ENTER STARTING VALUE OF W FOR CLUTTER & AMBIGUITY FCN ",WS
ACCEPT "ENTER FINAL VALUE OF W FOR CLUTTER & AMBIGUITY FCN ",WF
ACCEPT "ENTER INTERVAL BETWEEN W'S ",WIN
NUMW=ANINT((WF-WS)/WIN)
AREA=WIN*(TOTAL/M)
WRITE(10,300) WS,WF
WRITE(10,305) WIN,NUMW
WRITE(10,310) AREA
MAXVOL=0.0
MINVOL=0.0
AVGVOL=0.0
TYPE 'IF YOUR CLUTTER CAN NOT BE MODELED BY UNIFORM OR GAUSSIAN'
TYPE 'DISTRIBUTIONS IN DELAY & DOPPLER, THEN YOU MUST ENTER THE'
TYPE 'DISTRIBUTION POINT-BY-POINT.'

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ACCEPT 'ENTER 0 IF YOU INPUT CLUTTER DIST POINT-BY-POINT, ELSE 1', IP
ACCEPT 'ENTER LOWER BOUND ON TAU FOR CLUTTER ', CTS
ACCEPT 'ENTER UPPER BOUND ON TAU FOR CLUTTER ', CTF
IF (IP EQ. 1) GO TO 10
5 CALL IDC(TOTAL, WS, WF, N, DCWF, DCWS, DCTS, DCTF)
CSUM=0.0
GO TO 15
10 TYPE 'ENTER 0 IF CLUTTER UNIFORMLY DISTRIBUTED IN DELAY, 1 IF'
ACCEPT 'DISTRIBUTED AS GAUSSIAN', ICDY
TYPE 'ENTER 0 IF CLUTTER IS DISTRIBUTED UNIFORMLY IN DOPPLER, '
ACCEPT '1 IF DISTRIBUTED AS GAUSSIAN', ICDP
CALL CPDID(ICDY, ICDP, WMEAN, WSTDV, TMEAN, TSTDV)
15 DO 90 IA=1, NUMW
W=WS+WIN*(IA-1)
WRITE(10, 315) W
VALUE=CMPLX(0.0, W)
CALL SETUP(Z, FREQ, XVALS, YVALS, VALUE, TOTAL, SAMPLE, M)
CALL DFT5(XVALS, SAMPLE, 0)
CALL DFT5(YVALS, SAMPLE, 0)
DO 20 I=1, SAMPLE
XVALS(I)=CONJG(XVALS(I))
YVALS(I)=CONJG(YVALS(I))
XVALS(I)=XVALS(I)*YVALS(I)/SAMPLE
20 CONTINUE
CALL DFT5(XVALS, SAMPLE, 1)
DO 25 I=1, SAMPLE
MAG(I)=CABS(XVALS(I))*SCALE
25 CONTINUE
DO 35 I=1, SAMPLE
IF (I GE. M+1) GO TO 30
TIME=(TOTAL/M*(I-1))-TOTAL
GO TO 35
30 TIME=TOTAL/M*(I-M-1)
35 IF (IP NE. 0) GO TO 55
DO 50 K=1, N
IF ((W GE. DCWS(K)). AND. (W LE. DCWF(K))) GO TO 40
DENSITY=0.0
GO TO 50
40 IF ((TIME GE. DCTS(K)). AND. (TIME LE. DCTF(K))) GO TO 45
DENSITY=0.0
GO TO 50
45 WRITE(10, 340) W, TIME
ACCEPT 'ENTER VALUE OF CLUTTER DENSITY AT THIS POINT ', DENSITY
CSUM=CSUM+DENSITY
IF (CSUM LE. 1.0) GO TO 50
TYPE 'EXCEEDED LIMIT FOR CLUTTER DISTRIBUTION. . . ABORT AND START AGAIN'
50 CONTINUE
GO TO 60
55 CALL PROB(DENSITY, TIME, W, WS, WF, CTS, CTF, WMEAN, WSTDV, TMEAN, TSTDV, ICDY,
* ICDP)
60 PROD(I, 1)=MAG(I)*MAG(I)*DENSITY
65 CONTINUE
IF (IA EQ. 1) GO TO 83
DO 90 I=1, SAMPLE-1

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SUM=0.0
MAX=PROD(I,1)
MIN=PROD(I,1)
DO 75 K=1,2
DO 70 L=I,I+1
IF (PROD(L,K) LT MIN) MIN=PROD(L,K)
IF (PROD(L,K) GT MAX) MAX=PROD(L,K)
SUM=SUM+PROD(L,K)
70 CONTINUE
75 CONTINUE
AVG=SUM/4.0
MAXVOL=MAXVOL+MAX*AREA
MINVOL=MINVOL+MIN*AREA
AVGVOL=AVGVOL+AVG*AREA
80 CONTINUE
WRITE(10,320) IA
WRITE(10,400) IA, MAXVOL, MINVOL, AVGVOL
83 DO 85 I=1, SAMPLE
PROD(I,2)=PROD(I,1)
85 CONTINUE
90 CONTINUE
WRITE(10,325) MINVOL
WRITE(10,330) MAXVOL
WRITE(10,335) AVGVOL
RETURN
300 FORMAT('STARTING VALUE OF W=',F12.2,3X,'FINAL VALUE OF W=',F12.2)
305 FORMAT('INTERVAL BETWEEN W IS=',F12.2,3X,'# OF W TO BE USED= ',I5)
310 FORMAT('AREA=',F14.8)
315 FORMAT('W NOW BEING CONSIDERED=',F12.2)
320 FORMAT('NOW COMPLETED CALCULATIONS FOR W INTERVAL=',I5)
325 FORMAT('THE MINIMUM VOLUME OF THE INTEGRATED PRODUCT IS ',E10.4)
330 FORMAT('THE MAXIMUM VOLUME OF THE INTEGRATED PRODUCT IS ',E10.4)
335 FORMAT('THE AVERAGE VOLUME OF THE INTEGRATED PRODUCT IS ',E10.4)
340 FORMAT('W=',F12.2,3X,'TAU=',F12.2)
400 FORMAT('IA=',I4,2X,'MAX=',E10.4,2X,'MIN=',E10.4,2X,'AVG=',E10.4)
END

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C*****
C*      SUBROUTINE CPDID      *
C*      WRITTEN BY: CAPT JOHN REED      *
C*      15 APRIL 1982      *
C*****

C      'CPDID' IS A FORTRAN V SUBROUTINE WHICH DETERMINES THE STATISTICS
C      NEEDED TO DEFINE A STANDARD PROBABILITY DENSITY FUNCTION OF CLUTTER
C
C      'CPDID' MAY BE CALLED BY SUBROUTINE 'INTEG'.
C      'CPDID' RECEIVES THE FOLLOWING VARIABLES FROM 'INTEG': IP,ICDY,ICDP
C      'CPDID' RETURNS THE FOLLOWING VARIABLES TO 'INTEG':
C          WMEAN,WSTDV,TMEAN,TSTDV.
C
C      'CPDID' IS STORED IN FILE 'CPDID.FR' IN DIRECTORY DP4.REED.
C
C      'CPDID' IS COMPILED ON THE ECLIPSE USING THE COMMAND:
C          FORTRAN CPDID
C
C      THE FOLLOWING VARIABLES ARE USED IN 'CPDID':
C          IP - IDENTIFIES NON-STANDARD PROB DENSITY FCN
C          WMEAN - STATISTIC OF DOPPLER USED TO DEFINE DENSITY FCN
C          WSTDV - STATISTIC OF DOPPLER USED TO DEFINE DENSITY FCN
C          TMEAN - STATISTIC OF TAU USED TO DEFINE DENSITY FCN
C          TSTDV - STATISTIC OF TAU USED TO DEFINE DENSITY FCN
C          ICDY - IDENTIFIES DELAY PROB DENSITY FCN FOR CLUTTER
C          ICDP - IDENTIFIES DOPPLER PROB DENSITY FCN FOR CLUTTER

      SUBROUTINE CPDID(ICDY,ICDP,WMEAN,WSTDV,TMEAN,TSTDV)

      IF(ICDY EQ 1) GO TO 5
      TYPE 'CLUTTER IS DISTRIBUTED UNIFORMLY IN DELAY'
      GO TO 10
5      TYPE 'CLUTTER IS DISTRIBUTED GAUSSIANLY IN DELAY'
      ACCEPT 'ENTER THE MEAN OF DELAY',TMEAN
      ACCEPT 'ENTER THE STANDARD DEVIATION OF DELAY',TSTDV
10     IF(ICDP EQ 1) GO TO 15
      TYPE 'CLUTTER IS DISTRIBUTED UNIFORMLY IN DOPPLER'
      GO TO 20
15     TYPE 'CLUTTER IS DISTRIBUTED GAUSSAINLY IN DOPPLER'
      ACCEPT 'ENTER THE MEAN OF DOPPLER',WMEAN
      ACCEPT 'ENTER THE STANDARD DEVIATION OF DOPPLER',WSTDV
20     RETURN
      END

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C*****
C*      SUBROUTINE IDC      *
C*      WRITTEN BY: CAPT JOHN REED      *
C*      15 APRIL 1982      *
C*****

C      'IDC' IS A FORTRAN V SUBROUTINE WHICH DEFINES THE CLUTTER AREAS
C      (IN TERMS OF DOPPLER(W) & DELAY(TAU) FOR NON-STANDARD CLUTTER
C      DISTRIBUTIONS.
C
C      'IDC' MAY BE CALLED BY SUBROUTINE 'INTEG'
C      'IDC' RECEIVES THE FOLLOWING VARIABLES FROM 'INTEG':
C      TOTAL, WS, WF.
C      'IDC' RETURNS THE FOLLOWING VARIABLES TO 'INTEG':
C      N, DCWF, DCWS, DCTS, DCTF.
C
C      'IDC' IS COMPILED ON THE ECLIPSE USING THE COMMAND:
C      FORTRAN IDC
C
C      'IDC' IS STORED IN FILE 'IDC.FR' IN DIRECTORY DP4:REED.
C
C      THE FOLLOWING VARIABLES ARE USED IN 'IDC':
C      TOTAL - TOTAL TIME DURATION OF WAVEFORM
C      WS - STARTING VALUE OF DOPPLER FREQ FOR CLUTTER & AMB FCN
C      WF - FINAL VALUE OF DOPPLER FREQ FOR CLUTTER & AMB FCN
C      DCWF - ARRAY CONTAINING FINAL DOPPLER FREQ FOR EACH CLUTTER AREA
C      DCWS - ARRAY CONTAINING STARTING DOPPLER FREQ FOR EACH CLUTTER AREA
C      DCTS - ARRAY CONTAINING STARTING VALUE OF TAU FOR EACH CLUTTER AREA
C      DCTF - ARRAY CONTAINING FINAL TAU FOR EACH CLUTTER AREA
C      N - # OF DIFFERENT CLUTTER AREAS

      SUBROUTINE IDC(TOTAL, WS, WF, N, DCWF, DCWS, DCTS, DCTF)

      REAL DCWS(10), DCWF(10), DCTS(10), DCTF(10)

      ACCEPT 'ENTER # OF AREAS OF CLUTTER ', N
      DO 35 I=1, N
      ACCEPT 'ENTER STARTING VALUE OF W FOR NEXT CLUTTER AREA ', DCWS(I)
      ACCEPT 'ENTER FINAL VALUE OF W FOR THIS CLUTTER AREA ', DCWF(I)
      ACCEPT 'ENTER STARTING VALUE OF TAU FOR THIS CLUTTER AREA ', DCTS(I)
      ACCEPT 'ENTER FINAL VALUE OF TAU FOR THIS CLUTTER AREA ', DCTF(I)
      IF (DCWS(I) LT WS) GO TO 5
      GO TO 10
5      WRITE(10, 300) DCWS(I), WS
      DCWS(I)=WS
      IF (DCWS(I) GT DCWF(I)) DCWS(I)=DCWF(I)
10     IF (DCWF(I) GT WF) GO TO 15
      GO TO 20
15     WRITE(10, 305) DCWF(I), WF
      DCWF(I)=WF
      IF (DCWS(I) GT DCWF(I)) DCWS(I)=99999.
20     IF (DCTS(I) LT -TOTAL) GO TO 25
      GO TO 30
25     WRITE(10, 310) DCTS(I), -TOTAL

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DCTS(I)=-TOTAL
IF (DCTS(I).GT.DCTF(I)) DCTS(I)=DCTF(I)
30 IF (DCTF(I).LT.TOTAL) GO TO 35
WRITE(10,315) DCTF(I),TOTAL
DCTF(I)=TOTAL
IF (DCTS(I).GT.DCTF(I)) DCTS(I)=99999.
35 CONTINUE
RETURN

```

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300 FORMAT('DCWS(I)=' ,F12.2,5X,'LESS THAN WS, THUS MAKING DCWS(I)=' ,F12.2)
305 FORMAT('DCWF(I)=' ,F12.2,5X,'EXCEEDS WF, THUS MAKING DCWF(I)=' ,F12.2)
310 FORMAT('DCTS(I)=' ,F12.10,5X,'LESS THAN -TOTAL, MAKING DCTS(I)=' ,F12.10)
315 FORMAT('DCTF(I)=' ,F12.10,5X,'EXCEEDS TOTAL, MAKE DCTF(I)=' ,F12.10)
END

```

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C*****
C*      SUBROUTINE PROB      *
C*      WRITTEN BY: CAPT JOHN REED      *
C*      15 APRIL 1982      *
C*****

C      'PROB' IS A FORTRAN V SUBROUTINE WHICH DETERMINES THE VALUE OF
C      THE PROBABILITY DENSITY FUNCTION OF CLUTTER FOR A GIVEN DELAY(TAU)
C      AND DOPPLER FREQUENCY (W). THIS SUBROUTINE APPLIES ONLY FOR A
C      STANDARD PROBABILITY DENSITY FUNCTION.
C
C      'PROB' IS CALLED BY THE SUBROUTINE 'INTEG'
C      'PROB' RECEIVES THE FOLLOWING VARIABLES FROM 'INTEG':
C      TIME, W, ICDY, ICDP, WS, WF, CTS, CTF, WMEAN, WSTDV, TMEAN, TSTDV.
C      'PROB' RETURNS THE VARIABLE 'DENSITY' TO 'INTEG'.
C
C      'PROB' IS COMPILED ON THE ECLIPSE USING THE COMMAND:
C      FORTRAN PROB
C
C      'PROB' IS STORED IN FILE 'PROB FR' IN DIRECTORY DP4 REED.
C
C      THE FOLLOWING VARIABLES ARE USED IN 'PROB':
C      TIME - VALUE OF DELAY(TAU) FOR WHICH DENSITY IS CALCULATED
C      W - DOPPLER FREQ FOR WHICH DENSITY IS CALCULATED
C      CTS - STARTING VALUE OF TAU FOR CLUTTER
C      CTF - FINAL VALUE OF TAU FOR CLUTTER
C      IP - IDENTIFIES NON-STANDARD PROB DENSITY FCN
C      WF - FINAL VALUE OF DOPPLER FREQ FOR CLUTTER
C      WS - STARTING VALUE OF DOPPLER FREQ FOR CLUTTER
C      DENSITY-VALUE OF THE PROB DENSITY FCN FOR GIVEN W & TAU
C      TMEAN - STATISTIC OF TAU USED TO DESCRIBE PROB DENSITY FCN
C      TSTDV - STATISTIC OF TAU USED TO DESCRIBE PROB DENSITY FCN
C      WMEAN - STATISTIC OF W USED TO DESCRIBE PROB DENSITY FCN
C      WSTDV - STATISTIC OF W USED TO DESCRIBE PROB DENSITY FCN
C      SCALE - SCALING FACTOR OF DENSITY
C      ICDY - IDENTIFIES DELAY PROB DENSITY FCN FOR CLUTTER
C      ICDP - IDENTIFIES DOPPLER PROB DENSITY FCN FOR CLUTTER
C      DELAYDEN- VALUE OF DELAY PROB DENSITY FCN FOR GIVEN TAU
C      DOPDEN- VALUE OF DOPPLER PROB DENSITY FCN FOR GIVEN W
C
C      SUBROUTINE PROB(DENSITY, TIME, W, WS, WF, CTS, CTF, WMEAN, WSTDV,
C      *      TMEAN, TSTDV, ICDY, ICDP)
C
C      IF ((TIME LT CTS) OR (TIME GT CTF)) GO TO 25
C      IF (ICDY EQ 1) GO TO 5
C      DELAYDEN=1.0/(CTF-CTS)
C      GO TO 10
5      SCALE=1.0/SQRT(2.0*3.1416*WSTDV*TSTDV)
      AMT1=(TIME-TMEAN)/TSTDV
      DELAYDEN=SCALE*EXP(-0.5*AMT1*AMT1)
10     IF (ICDP EQ 1) GO TO 15
      DOPDEN=1.0/(WF-WS)
      GO TO 20

```



15 SCALE=1.0/SQRT(2.0\*3.1416\*WSTDV\*WSTDV)  
AMT2=(W-WMEAN)/WSTDV  
DOPDEN=SCALE\*EXP(-0.5\*AMT2\*AMT2)  
20 DENSITY=DELAYDEN\*DOPDEN  
GO TO 30  
25 DENSITY=0.0  
30 RETURN  
END

```

C*****
C*      PROGRAM CLUTR      *
C*      PRODUCED BY CAPT JOHN REED      *
C*      21 MAY 1982      *
C*****
C 'CLUTR' IS A FORTRAN V PROGRAM WHICH DETERMINES THE VALUES OF
C THE PROBABILITY DENSITY FUNCTION OF CLUTTER AS A FUNCTION OF DELAY
C AND DOPPLER. THE USER SPECIFIES THE DENSITY FUNCTION (CAN BE
C UNIFORM OR GAUSSIAN) FOR BOTH DELAY AND DOPPLER. SINCE THE VARIABLES
C ARE CONSIDERED INDEPENDENT, THE CLUTTER PROB DENSITY FCN WILL BE
C THE PRODUCT OF THE DELAY AND DOPPLER DENSITY FUNCTIONS. IT STORES
C QUANTIZED SAMPLES OF THE PROB DENSITY FCN IN FILE 'CLUTTER' WHICH
C CAN BE GRAPHED USING PROGRAM 'PLTTRNS'.
C
C 'CLUTR' MAY CALL THE SUBROUTINE 'COMP'. IF SO, IT PASSES THE
C VARIABLES TAU, TMEAN, TSTDV (OR W, WMEAN, WSTDV) TO 'COMP' AND
C RECEIVES THE VARIABLE DEL(OR DOP) FROM 'COMP'.
C
C 'CLUTR' IS COMPILED ON THE ECLIPSE USING THE COMMAND:
C      FORTRAN CLUTR
C
C 'CLUTR' IS STORED IN FILE 'CLUTR.FR' IN DIRECTORY DP4:REED.
C
C THE FOLLOWING VARIABLES ARE USED IN 'CLUTR':
C      TAU - DELAY VALUE
C      IDT - IDENTIFIES TYPE OF DELAY DISTRIBUTION
C      IDD - IDENTIFIES TYPE OF DOPPLER DISTRIBUTION
C      TMEAN - MEAN VALUE OF DELAY IF GAUSSIAN DISTRIBUTED
C      TSTDV - STANDARD DEVIATION OF DELAY IF GAUSSIAN DISTRIBUTED
C      WMEAN - MEAN VALUE OF DOPPLER IF GAUSSIAN DISTRIBUTED
C      WSTDV - STANDARD DEVIATION OF DOPPLER IF GAUSSIAN DISTRIBUTED
C      W - DOPPLER VALUE
C      DELDEN- VALUE OF DELAY DENSITY FCN FOR A GIVEN DELAY
C      DOPDEN- VALUE OF DOPPLER DENSITY FCN FOR A GIVEN DOPPLER
C      DENSITY-VALUE OF CLUTTER PROB DENSITY FCN FOR A GIVEN DELAY & DOPPLER
C      GRAPH - ARRAY OF QUANTIZED SAMPLES OF THE CLUTTER PROB DENSITY FCN
C      STORE - COMPLEX ARRAY OF GRAPH FOR A GIVEN DOPPLER

REAL DENSITY(64,64), MAX
COMPLEX STORE(64)
INTEGER GRAPH(64,64)

CALL DFILW('CLUTTER', IER)
TYPE 'FOR DFILW, IER=', IER
CALL CFILW('CLUTTER', 2, IER)
TYPE 'FOR CFILW, IER=', IER
CALL OPEN(6, 'CLUTTER', 2, IER)
TYPE 'FOR OPEN, IER=', IER
ACCEPT 'ENTER STARTING VALUE OF DELAY ON GRAPH ', GSTART
ACCEPT 'ENTER FINAL VALUE OF DELAY ON GRAPH ', GSTOP
ACCEPT 'ENTER STARTING VALUE OF DELAY ', TSTART
ACCEPT 'ENTER FINAL VALUE OF DELAY ', TSTOP
ACCEPT 'ENTER STARTING VALUE OF DOPPLER ', WSTART

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ACCEPT 'ENTER FINAL VALUE OF DOPPLER ',WSTOP
TINTERVAL=(GSTOP-GSTART)/64.0
WINTERVAL=(WSTOP-WSTART)/64.0
ACCEPT 'ENTER 0 IF DELAY DISTRIBUTED UNIFORMLY, 1 IF GAUSSIAN ',IDT
IF (IDT EQ 1) GO TO 5
TYPE 'DELAY DISTRIBUTED UNIFORMLY'
GO TO 10
5 TYPE 'DELAY DISTRIBUTED AS GAUSSIAN'
ACCEPT 'ENTER MEAN VALUE OF DELAY ',TMEAN
ACCEPT 'ENTER STANDARD DEVIATION OF DELAY ',TSTDV
10 ACCEPT 'ENTER 0 IF DOPPLER DISTRIBUTED UNIFORMLY, 1 IF GAUSSIAN ',IDD
IF (IDD EQ 1) GO TO 15
TYPE 'DOPPLER DISTRIBUTED UNIFORMLY'
GO TO 20
15 TYPE 'DOPPLER DISTRIBUTED AS GAUSSIAN'
ACCEPT 'ENTER MEAN VALUE OF DOPPLER ',WMEAN
ACCEPT 'ENTER STANDARD DEVIATION OF DOPPLER ',WSTDV
20 MAX=0.0
DO 50 I=1,64
W=WSTART+(I-1)*WINTERVAL
DO 45 J=1,64
TAU=GSTART+(J-1)*TINTERVAL
IF (TAU LT. TSTART OR. TAU GT. TSTOP) GO TO 29
IF (IDT EQ 1) GO TO 25
DELDEN=1.0/(TSTOP-TSTART)
GO TO 30
25 CALL COMP(TAU,TMEAN,TSTDV,DEL)
DELDEN=DEL
GO TO 30
29 DELDEN=0.0
30 IF (IDD EQ 1) GO TO 35
DOPDEN=1.0/(WSTOP-WSTART)
GO TO 40
35 CALL COMP(W,WMEAN,WSTDV,DOP)
DOPDEN=DOP
40 DENSITY(J,I)=DELDEN*DOPDEN
IF (DENSITY(J,I) GT. MAX) MAX=DENSITY(J,I)
45 CONTINUE
50 CONTINUE
DO 65 I=1,64
DO 60 J=1,64
K=0
VALUE=DENSITY(J,I)/MAX
DO 55 L=1,200
R=0.005*L
IF (VALUE GE. R) K=L
55 CONTINUE
GRAPH(J,I)=K
60 CONTINUE
65 CONTINUE
DO 75 I=1,64
DO 70 J=1,64
STORE(J)=GRAPH(J,I)
70 CONTINUE

```

```

75      CALL WRBLK(6, I, STORE, I, IER)
        CONTINUE
        CALL CLOSE(6, IER)
        TYPE 'FOR CLOSE, IER=', IER
        WRITE(10, 300) TSTART, TSTOP
        WRITE(10, 305) WSTART, WSTOP
        WRITE(10, 310) MAX
        TYPE "TO GET DISPLAY, RUN 'PLTTRNS CLUTTER/I 64/N' ON TEXTRONIX"
300      FORMAT('TAU STARTS AT ', F12.9, 5X, 'TAU STOPS AT ', F12.9)
305      FORMAT('DOPPLER STARTS AT ', F12.3, 5X, 'DOPPLER STOPS AT ', F12.3)
310      FORMAT('MAXIMUM VALUE OF CLUTTER DISTRIBUTION=', F12.6)
        END

```

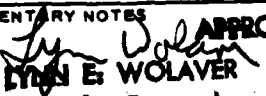
## VITA

John David Reed was born on 25 April 1952 in Albertville, Alabama. He graduated from Arab High School in Arab, Alabama in 1970. He attended the University of Alabama majoring in mathematics and minoring in chemistry. Upon his graduation on 12 May 1974, he received his B.S. degree and his commission from the U.S. Air Force through the Reserve Officer Training Corps. He attended the Electronic Systems Officer Course at Keesler AFB, Mississippi from May to December 1974. He was next assigned to the 4754th Radar Evaluation Squadron, Hill AFB, Utah, where he served as a Radar Evaluation Officer. He was then assigned to the 20th Surveillance Squadron, Eglin AFB, Florida, and served as the Maintenance Control Supervisor. In August 1979 he was assigned to the School of Engineering, Air Force Institute of Technology, to pursue a B.S.E.E. degree. Upon his graduation with a B.S.E.E. in March 1980, Captain Reed was selected to remain at the Air Force Institute of Technology to pursue an M.S.E.E. with emphasis in Communications/Radar and Digital Systems. He is a member of Phi Eta Sigma, Tau Beta Pi, and Eta Kappa Nu. He is happily married to the former Marianne Helquist of Northglenn, Colorado, and is the father of two children. The author's permanent address is:

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Block 20. ABSTRACT

analysis of the magnitude of the ambiguity function, signal-to-clutter ratio, and other basic principles of radar theory.

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